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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

SEDIMENTATION IN QUIESCENT AND TURBULENT BASINS

By J. J. Slade[1], Jr., Esq.

Synopsis

In a paper which has become a classic[2] the late Allen Hazen, M. Am. Soc. C. E., stated the problem of sedimentation with characteristic clearness. For sediment consisting of particles of constant hydraulic value in a completely turbulent basin (the term will be defined subsequently) Hazen's theory is complete; but actual sediment is usually far from uniform, and, therefore, Hazen's formulas are not always applicable.

The purpose of this paper is to continue the analysis and to develop a workable theory which takes into account the variation in hydraulic value of the consituent elements of actual sediment as well as the variation in the degree of turbulence of the basin. The procedure followed herein is such that the formulas obtained may be generalized to any desired degree; that is, only practicability need limit the number of constants entering into them and the consequent flexibility of the curves representing the processes.

The constants required will be found to fall into three distinct classes: (1) Those which represent the characteristics of the sediment; (2) those which represent the characteristics of the settling basin; and (3) those which depend on both the character of the sediment and the degree of agitation of the fluid. The separation of the constants into these three categories will, it is hoped, suggest experimental procedures for the study of settling-tank characteristics and be an aid in the understanding of this complex phenomenon.

Notation.—The symbols used in this paper are introduced as they occur and are presented, for convenience of reference, in the Appendix.

The Quiescent Basin

It is necessary to investigate thoroughly the mechanics of sedimentation in the still basin before attempting to analyze the phenomenon in the turbulent

Note.—Discussion on this paper will be closed in March, 1936, *Proceedings*.

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[2] "On Sedimentation", *Transactions*, Am. Soc. C. E., Vol. LIII (1904), pp. 43–88.

tank. Assume, then, a tank of still water of constant depth, h, which holds in suspension a quantity, B, of solid matter, which has a constant hydraulic value, v. Assume also that at the beginning of the process the solid matter is uniformly distributed throughout the liquid and that there is a quantity, b, per unit of volume. The time required for a particle to fall from a height, y, to the bottom of the tank is:

$$t_y = \frac{y}{v} \quad \ldots\ldots (1)$$

and the time required for a particle to fall from the surface is:

$$t = \frac{h}{v} \quad \ldots\ldots (2)$$

Eliminating v between Equation (1) and Equation (2), the following relation is derived:

$$y = \frac{h t_y}{t} \quad \ldots\ldots (3)$$

The quantity of sediment that falls to the bottom in time, $a = t_y$, is $b\ y\ A$ (in which A is the horizontal area of the tank), and this is equal to $\frac{b\,A\,h\,a}{t}$, using the value obtained for y in Equation (3). Since $b\ A\ h = B$, however, the quantity of solid matter remaining in suspension at the end of time, a, is:

$$B_a = B\left(1 - \frac{a}{t}\right) \quad \ldots\ldots (4)$$

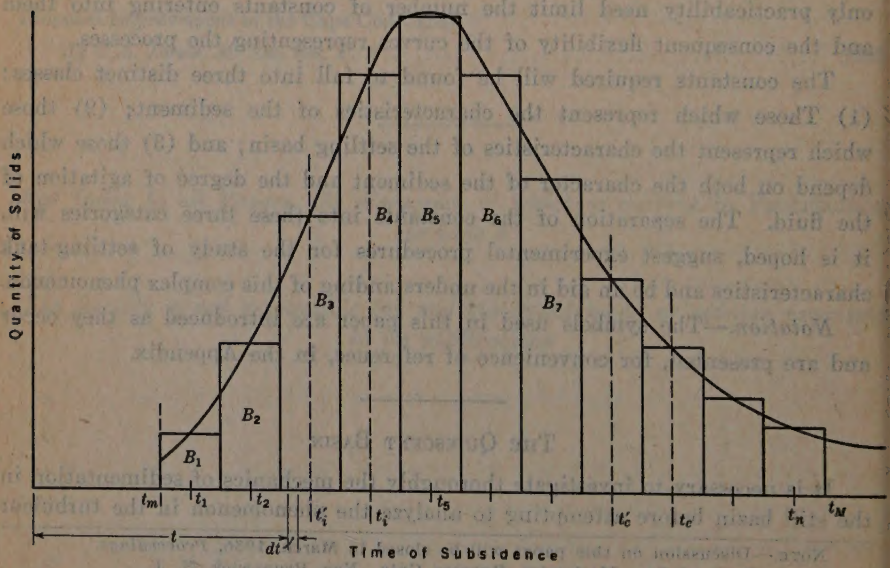

FIG. 1.

This is Hazen's first formula. It is of fundamental importance, all subsequent expressions being obtained from it by means of integrations and summations.

Assume now that, instead of a constant hydraulic value, the constituent particles have values, v, ranging continuously from a minimum, v_M, to a maximum, v_m, and let t be the time it takes the particles of hydraulic value, v, to fall from the surface to the bottom of the tank; t will then range from a minimum, t_m, to a maximum, t_M, since $t = \frac{h}{v}$. In general, it will not be known what proportion of the suspended solid matter has a given hydraulic value; but this unknown distribution function of the times of subsidence may be designated as $\phi(t)$; thus, $\phi(t)\ dt$ is the quantity of suspended matter having times of subsidence between t and $t + dt$; or, what amounts to the same thing, having hydraulic values lying between v and $v + dv$ (see Fig. 1). Before attempting to determine this function it is well to note some of its formal properties.

If it is required to find the quantity of sediment, dB_a, remaining in suspension at the end of time, dt, the hydraulic values of which lie between v and $v + dv$, Equation (4) and the definition of the distribution function yield:

$$dB_a = \phi(t)\left(1 - \frac{a}{t}\right)dt \qquad (5)$$

The total quantity remaining at the end of time, a, will be the integral of this expression; that is:

$$B_a = \int_{p(a)}^{t_M} \phi(t)\left(1 - \frac{a}{t}\right) dt \qquad (6)$$

The integration, being with respect to the times of subsidence, must range from t_m to t_M. The upper limit of this integral is certainly t_M. At first, the lower limit must be t_m, because particles of all hydraulic values are falling; but after a time, $a = t_m$, those particles of greatest hydraulic value will all have fallen. In general, at the end of time, $a = t_1$, say, all particles of hydraulic values greater than v_1 will have fallen to the bottom of the tank. Consequently, after a time of settling, $a = t_m$, the lower limit of the integral must be a.

Referring to Fig. 2, let $p(a)$ equal t_m for all values of a less than t_m; and a for all values of a between t_m and t_M. Then Equation (6) as it stands will give the quantity of suspended solids remaining at the end of time, a, for all

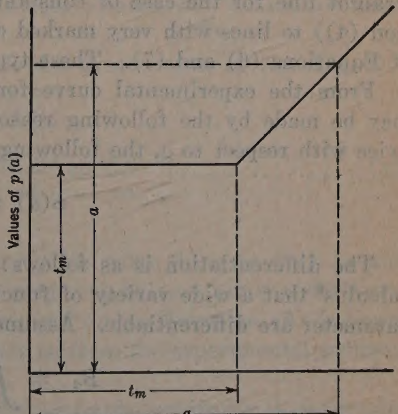

FIG. 2.

values of a from 0 to t_M, when all the solids will have settled. This formula applies, of course, to the settleable solids.

What is considered settleable and colloidal solids is probably an arbitrary, although convenient, classification. In practice, some upper limit for the time of subsidence, t_M, might be set, particles having a greater time of subsidence than this being classed with the colloids. If the quantity of this colloidal and near-colloidal material is B_c, Equation (6) becomes:

$$B_a = \int_{p(a)}^{t_M} \phi\,(t) \left(1 - \frac{a}{t}\right) dt + B_c \ldots\ldots\ldots\ldots (7)$$

From the definition of the distribution function it is also to be noted that:

$$\int_{t_m}^{t_M} \phi(t)\, dt = B - B_c = B_s \ldots\ldots\ldots\ldots (8)$$

Equation (8) merely states that the sum of all particles of all hydraulic values from v_m to v_M at the beginning of the process (that is, when $a = 0$) is equal to the quantity of settleable material.

The Distribution Function

In order to make use of Equations (6) and (7) as they stand it would be necessary to know the form of the function, $\phi(t)$, for the sediment in question, and, by experiment, this would be difficult. The direct procedure would consist in sorting the settleable solids according to size and applying Stokes' law (which gives the relation between size of particles and their hydraulic values) to the various classes found. The quantity in each class, plotted against the time of sedimentation thus found, would give $\phi(t)$.

It will now be shown, however, that $\phi(t)$ may be determined easily by using its formal properties, together with the experimental curve for B_a. This experimental sedimentation curve is readily determined; in fact, its determination constitutes a large part of the experimental work that has been done in the past on sedimentation investigations. Its shape varies from a straight line for the case of constant hydraulic value (represented by Equation (4)) to lines with very marked curvature given by formulas of the types of Equations (6) and (7). These types are sketched in Fig. 3.

From the experimental curve for B_a a graphical determination of $\phi(t)$ may be made by the following reasoning: If Equation (7) is differentiated twice with respect to a, the following relation is obtained,

$$\phi(a) = a \frac{d^2 B_a}{da_2} \ldots\ldots\ldots\ldots (9)$$

The differentiation is as follows: It is shown in treaties on the integral calculus[8] that a wide variety of functions defined by definite integrals with a parameter are differentiable. Assume that,

$$B_a = \int_{g(a)}^{f(a)} F(a, t)\, dt \ldots\ldots\ldots\ldots (10)$$

[8] See, for instance, Woods, "Advanced Calculus", Ginn & Co., 1926, pp. 141–143; also, Granville, "Differential and Integral Calculus", Ginn & Co., 1911, pp. 22–23.

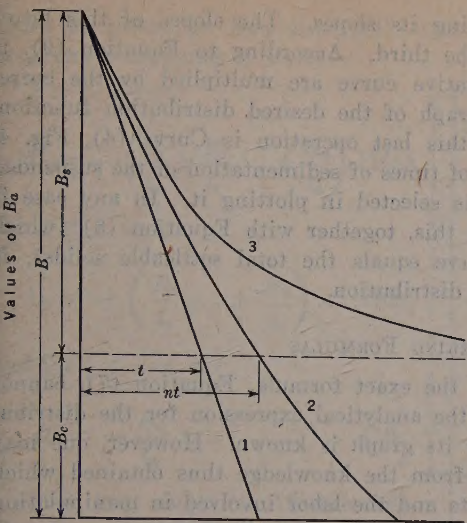

FIG. 3.

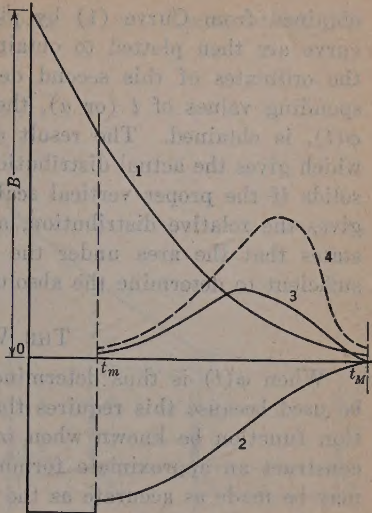

FIG. 4.

in which the limits are functions of the parameter, a, then the formula for differentiation is:

$$\frac{dBa}{da} = \int_{g(a)}^{f(a)} \frac{\partial F(a, t)}{\partial a} dt + F(a, f(a)) \frac{df(a)}{da} + F(a, g(a)) \frac{dg(a)}{da} \quad ..(11)$$

Applying Equation (11) to Equation (6) once:

$$\frac{dB_a}{da} = - \int_{p(a)}^{t_M} \frac{\phi(t)}{t} dt - \phi(a) \left[1 - \frac{a}{p(a)} \right] \frac{dp(a)}{da} \quad ....(12)$$

in which the middle term has disappeared because the derivative of the upper limit is zero. Now, the last term also vanishes because $\frac{dp(a)}{da} = 0$ for values of a less than t_m, and $\frac{a}{p(a)} = 1$ for values of a greater than t_m (see Fig. 2).

Applying Equation (11) to Equation (12):

$$\frac{d^2B_a}{da^2} = \frac{\phi[p(a)]}{p(a)} \frac{dp(a)}{da} = \frac{\phi(a)}{a} \quad ....(13)$$

which is Equation (9).

It is to be noted first that $\phi(a)$ is exactly the same thing as $\phi(t)$, because a and t assume the same values in the range, t_m to t_M. Since a derivative curve is the graph of the slopes of the original curve, Equation (9) indicates how to find $\phi(t)$ from the graph of B_a; that is, from the experimental sedimentation curve.

In Fig. 4, Curve (1) shows the quantity of solids remaining in suspension plotted against the time, as determined experimentally. Curve (2) has been

obtained from Curve (1) by plotting its slopes. The slopes of this latter curve are then plotted to obtain the third. According to Equation (9), if the ordinates of this second derivative curve are multiplied by the corresponding values of t (or a), the graph of the desired distribution function, $\phi(t)$, is obtained. The result of this last operation is Curve (4), Fig. 4, which gives the actual distribution of times of sedimentation of the suspended solids if the proper vertical scale is selected in plotting it. In any case it gives the relative distribution, and this, together with Equation (8) (which states that the area under the curve equals the total settleable solids), is sufficient to determine the absolute distribution.

The Working Formulas

When $\phi(t)$ is thus determined, the exact formula, Equation (7), cannot be used because this requires that the analytical expression for the distribution function be known when only its graph is known. However, one may construct an approximate formula from the knowledge thus obtained which may be made as accurate as the data and the labor involved in manipulating it will warrant.

To obtain this approximate equation break up the distribution function into a convenient number of parts, say n, as shown in Fig. 1, and take the average hydraulic value of each part. Equation (7) then becomes:

$$B_a = \sum_{r=1}^{n} B_r \left(1 - \frac{a}{t_r}\right) + B_c \quad \text{.................. (14)}$$

in which B_r is the quantity of sediment with an average hydraulic value, v_r; t_r is the corresponding time of subsidence.

Attention is called to the fact that when $a = t_1$ all the sediment with a hydraulic value of v_1 has fallen to the bottom and that, therefore, in Equation (14) the first term appears only for values of a less than t_1. As a increases past the value of each t_r, each term of Equation (14) drops out, successively. This progressive dropping of terms corresponds to the variable lower limit in the exact Equation (7). Obviously,

$$\sum_{r=1}^{n} B_r = B_s \quad \text{........................ (15)}$$

This is merely the approximate form of Equation (8).

Discontinuous Distributions

It may happen that the sediment does not have a continuous distribution of hydraulic values, but that there are n kinds of particles of quantities, B_1, B_2, ..., B_n, with corresponding hydraulic values, v_1, v_2, ..., v_n. The experimental curve will then take the form of the broken line, Curve (1) of Fig. 5. In this case Equation (14) is the exact form, and the distribution function is the set of B-values. To obtain these quantities first differen-

tiate B_a given by Equation (14) with respect to a. Remembering the progressive dropping of terms:

For $a < t_1$:

$$D_1 = -\left(\frac{B_1}{t_1} + \frac{B_2}{t_2} \dots + \frac{B_n}{t_n}\right) \dots (16a)$$

for $t_1 < a < t_2$,

$$D_2 = -\left(\frac{B_2}{t_2} \dots + \frac{B_n}{t_n}\right) \dots (16b)$$

and for $t < a < t_n$:

$$D_n = -\frac{B_n}{t_n} \dots\dots\dots\dots (16c)$$

The step function (Curve (2), Fig. 5) is the result of plotting these quantities; that is, it is the graph of the slopes of the broken line, Curve (1), Fig. 5. The differences of every two successive values of D are expressed as:

$$D_2 - D_1 = \frac{B_1}{t_1} \dots\dots (17a)$$

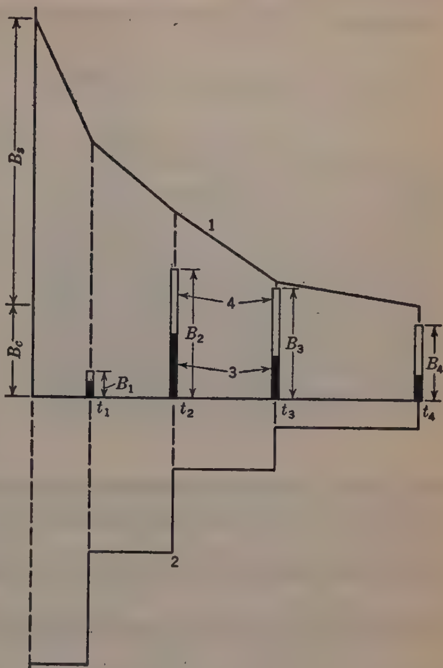

FIG. 5.

$$D_3 - D_2 = \frac{B_2}{t_2}, \text{ etc.} \dots\dots\dots\dots (17b)$$

or, generally,

$$B_r = t_r \; (D_{r+1} - D_r) \dots\dots\dots\dots (18)$$

in which $r = 1, 2, \dots, n$.

Equation (18) corresponds exactly to Equation (9), so that, although the step function given by Equation (16) cannot be properly said to possess a derivative other than zero, still the differences of the ordinates at the points of discontinuity are equal to the quantities, B, when multiplied by the corresponding values of t. These differences are shown by Ordinates 3 of Fig. 5, while Ordinates 4, obtained from Ordinates 3 by multiplying by the corresponding abscissas, represent the discontinuous distribution function; that is, they represent the set of Q-values.

STRATIFICATION OF SEDIMENT

It may be well to note here that quiescent sedimentation is characterized by the stratification that will occur in the upper regions of the basin. At the end of time, a, there will be a clear layer (except for fine colloids) of depth, $v_M\, a$, as shown in Fig. 6. From that level down to a depth, $v_m\, a$, the

suspended material will be found in increasing density (in layers if the distribution of hydraulic values is discontinuous, and gradual if the distribution is continuous). The vertical distribution of the density of suspended matter

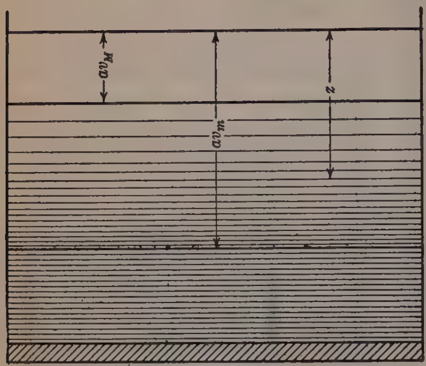

FIG. 6.

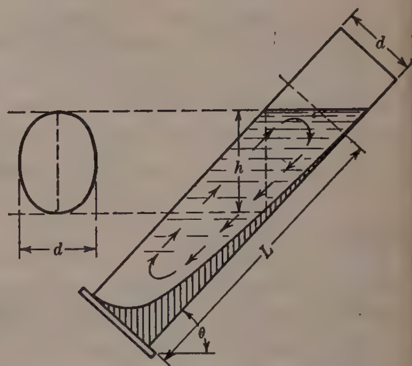

FIG. 7.

in the liquid will be proportional to B_a in Equation (7) or Equation (14), as the case may be, because at a depth, z, below the surface there will be found only material with a hydraulic value less than $\frac{z}{a}$. At a depth greater than $v_m a$, the density will be what it was throughout the basin originally.

THE TURBULENT TANK

The foregoing analysis has been developed for the purpose of determining the hydraulic properties of the solid material suspended in the liquid of the settling tank, which is seldom still. The important problem of sedimentation is to determine the behavior of the suspended matter when a certain degree of turbulence is present in the tank.

Turbulence is defined herein as the average degree of agitation or circulatory motion of the water in the tank. It is conceivable that this turbulence might be defined as some function of the average velocity but, for the purpose of the following analysis, this correlation with velocity need not be made. Just as in the preceding development it was not necessary to determine the masses or sizes of the constituent particles, so now the turbulence in the tank will be considered only as it affects the hydraulic behavior of the suspended material.

As treated in this paper, therefore, turbulence is quite a relative term. A certain amount of circulation in a tank which would not affect, appreciably, the time required for grains of sand to subside, for instance, might be sufficient to pick silt from the bottom and keep it in suspension as long as the circulation persisted. From the writer's point of view this same circulation would present two distinct degrees of turbulence, in this example: One with respect to the sand; and the other with respect to the silt. The degree of turbulence

is a difficult concept, but it seems reasonable to make the following simplifications (an extension of this classification will no doubt be made necessary by the results of future experiments).

Assume, first, the suspended matter in a basin of water to be uniform and of constant hydraulic value, v. If a slight circulatory motion be imparted to the water, the first appreciable effect which this agitation will have will be to lengthen the time of subsidence of the material from t to kt, say. With respect to this material the motion at this stage is termed "incomplete turbulence", the only appreciable effect of which is to lengthen the time of subsidence. Except for this increase, sedimentation in a tank in which the water is only slightly turbulent will be identical with quiescent sedimentation.

Assume, next, that the agitation is gradually increased. A state of turbulence will eventually arise in which the phenomenon of sedimentation will be radically different from that of the preceding state. The material that remains in suspension will be mixed thoroughly so that its density is uniform throughout the basin, and that which settles will remain at the bottom. This is the state of turbulence required in Hazen's theory. Herein it will be termed "complete turbulence".

Finally, if the agitation increases, a state of turbulence will be reached in which the material will not settle. The solids that may be at the bottom will be picked up and held in suspension, thoroughly mixed. This condition will be termed "critical turbulence". In all likelihood, the transition from one state of turbulence to another will not be sharply defined, and careful experiments may show transition stages that need to be taken into account. In what follows, however, only these three conditions will be considered.

Thus far, turbulence has been defined for material of constant hydraulic value, and, consequently, for variable circulation of the liquid. If, as is the usual case, a material of variable hydraulic values and fairly uniform circulation of the water is encountered, the foregoing notions may be applied in the following manner: Assume the material to have hydraulic values ranging from v_m to v_M, and assume the water to have a given uniform and constant degree of agitation; then there will be a value, v_i, such that for all particles with hydraulic values greater than v_i the turbulence will be incomplete. There will also be a value, v_c, such that the turbulence will be complete for particles with hydraulic values between v_i and v_c. This same agitation will be critical for all particles with hydraulic values less than v_c.

If the material in a tank is distributed with respect to its times of subsidence, as shown in Fig. 1, then, for a given agitation of the water, there will be two values of t — t_i and t_c, say—such that the turbulence is incomplete, complete, and critical in the intervals, t_m to t_i, t_i to t_c, and t_c to t_M, respectively. For some other degree of agitation these transition points will be t'_i and t'_c.

Incomplete Turbulence

For the state of incomplete turbulence the formulas obtained for quiescent sedimentation will hold with slight modifications. The only alteration will occur in the time of sedimentation of the various particles, which will increase

according to some law, $k(t)$, t being the time of settling in a quiescent basin. This law must be determined experimentally. Assuming it to be known, Equation (5) becomes:

$$dB_a = \phi\,(t)\left[1 - \frac{a}{k\,(t)}\right] dt \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Equation (19) gives the quantity of sediment remaining in suspension at the end of the time in which the hydraulic values (in a quiet tank) lie within the interval, t and $t + dt$. For the total quantity of sediment remaining at the end of this time, instead of Equation (6) the formula is,

$$B_a = \int_{p(a)}^{t_M} \phi\,(t)\left[1 - \frac{a}{k\,(t)}\right] dt \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

If instead of the function, $k(t)$, a set of factors, k_1, k_2, ..., k_n, is determined which, for a given degree of agitation, gives the times of subsidence, $k_1\,t_1$, $k_2\,t_2$, ..., $k_n\,t_n$, of particles for which the times of subsidence in the still tank are t_1, t_2, ..., t_n, respectively; then the approximate Equation (7) becomes:

$$B_a = \sum_{r=1}^{n} B_r\left(1 - \frac{a}{k_r\,t_r}\right) \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

These factors, k_r, depend both on the hydraulic values of the particles and on the degree of turbulence in the tank.

Complete Turbulence

It is with the state of complete turbulence, as defined herein, that Hazen was chiefly concerned.[2] The characteristic feature of this state is that the turbulence is sufficient to keep the sediment uniformly distributed throughout the tank, but not sufficient to pick up that which has once fallen to the bottom.

To determine the quantity of sediment (assumed at present to have a constant hydraulic value, v) that remains in suspension at the end of time, a, it is proposed to follow Hazen's procedure in its essentials: The time, a, is divided into n equal parts, each interval, $\frac{a}{n}$, being so small that the water in the tank may be considered quiet during that time. At the end of the first interval, the quantity, B_1, of sediment remaining will be, by Equation (4),

$$B_{1a} = B\left(1 - \frac{a}{n\,t}\right) \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

At the end of the second interval the quantity remaining will be,

$$B_{2a} = B_1\left(1 - \frac{a}{n\,t}\right) = B\left(1 - \frac{a}{n\,t}\right)^2 \ldots\ldots\ldots\ldots (23)$$

Repeating the process n times the quantity at the end of time, a, will be:

$$B_a = B\left(1 - \frac{a}{n\,t}\right)^n \qquad (24)$$

This is, substantially, Hazen's second formula. This is the form in which it was presented and has since been retained, but it is awkward to apply. However, it attains a definite limit as the number, n, of divisions is made indefinitely great, which is, of course, the actual condition.

The base of natural logarithms is usually defined[3] as:

$$e = \underset{(m=0)}{\text{limit}}\ (1 + m)^{\frac{1}{m}} \qquad (25)$$

If $m = -\frac{a}{n\,t}$, Equation (24) becomes:

$$B_a = B\,(1 + m)^{-\frac{a}{m\,t}} = B\left[(1 + m)^{\frac{1}{m}}\right]^{-\frac{a}{t}} \qquad (26)$$

and, in the limit, n becomes infinite, m becomes 0, and the expression in brackets becomes e, Equation (25). Therefore (see Fig. 3, Curve (3)):

$$B_a = B\,e^{-\frac{a}{t}} \qquad (27)$$

If, as before, instead of material with constant hydraulic value, the water carries in suspension solids with times of subsidence distributed according to the function, $\phi(t)$, then the quantity of material remaining in suspension at the end of time, a, is:

$$B_a = \int_{t_m}^{t_M} \phi\,(t)\,e^{-\frac{a}{t}}\,dt \qquad (28)$$

It is to be noted that, in Equation (28), unlike the case of Equation (6), the lower limit is t_m throughout the process, because, since the turbulence is complete, none of the material ever settles completely. This expression does not yield a simple relation connecting the sedimentation curve with the distribution function analogous to that of Equation (9). It is for this reason that it is necessary to determine the function, $\phi(t)$, from the sedimentation curve for the quiescent state. The distribution function is a property of the sediment only, of course, and so is quite independent of the state of turbulence.

Mathematically, Equation (28) presents no great difficulty. By a slight transformation it becomes what is known as an integral equation of the Laplace-Fourier type. Such integral equations have wide application in physics and engineering. There are many useful relations known that exist between the sedimentation curve, B_a, and the distribution function, $\phi(t)$, connected as in Equation (28); but the writer has failed to find one with a simple graphical interpretation such as that of Equation (9).

From this formula (Equation (28)), an approximate formula is derived:

$$B_a = \sum_{r=1}^{n} B_r\,e^{-\frac{a}{t_r}} \qquad (29)$$

which is the exact form when the distribution is discontinuous.

Critical Turbulence

Material for which the turbulence in a settling tank is critical does not settle, because that which reaches the bottom is picked up again. For turbulence of this type, $B_a = B_c$, in which B_c is the material with time of subsidence greater than t_c as previously defined.

The Actual Basin

In any settling tank there will ordinarily be material with a wide range of hydraulic values, so that the normal circulation of the water in the basin will be turbulence of one or another of the types described herein for some part of the sediment. As has already been stated, a proportion of the sediment will have times of subsidence between t_m and t_i for which the turbulence will be incomplete, another portion with times of subsidence between t_i and t_c for which the turbulence will be complete and, finally, a part with times of subsidence greater than t_c for which the turbulence will be critical (see Fig. 1).

The constants, t_i and t_c, are characteristic of the tank. They measure the turbulence of the fluid in it. They are taken into account in the general formula for the settling basin by noticing that they become limits of integration. Combining all results, the general expression for the sediment remaining in suspension at the end of time, a, is:

$$B_a = \int_{p(a)}^{t_i} \phi(t) \left[1 - \frac{a}{k(t)} \right] dt + \int_{t_i}^{t_c} \phi(t)\, e^{-\frac{a}{t}}\, dt + B_c \quad \ldots\ldots (30)$$

From Equation (30) the approximate expression is obtained, which will be exact for the case of discontinuous distribution of hydraulic values, by dividing the range, t_m to t_c, into n parts, of which the first, s, will be in the range from t_m to t_i. Summing over these ranges:

$$B_a = \sum_{r=1}^{s} B_r \left(1 - \frac{a}{k_r\, t_r} \right) + \sum_{r=s+1}^{n} B_r\, e^{-\frac{a}{t_r}} + B_c \quad \ldots\ldots\ldots (31)$$

This general expression may be carried to any degree of refinement, but in engineering practice a very few terms should suffice. It exhibits at once the constants of the theory and their categories: The B-values which are characteristic of the sediment; the limits of summation (or integration), which separate the expression into three groups, corresponding to the three kinds of turbulence, and which are characteristic of the basin; and the k-values, which depend both on the basin and the sediment.

Examples

A. W. Dilling, Assoc. M. Am. Soc. C. E., and Langdon Pearse, M. Am. Soc. C. E., report[4] the results of certain experiments which may be used to illustrate the theory developed herein. Samples of sludge were settled in

[4] Rept. on Industrial Wastes from the Stockyards and Packingtown in Chicago, by A. W. Dilling and Langdon Pearse, Vol. II, January, 1921 (The Sanitary Dist. of Chicago), pp. 140–141.

glass cylinders and the sedimentation curves recorded. The cylinders were held upright and also inclined at various angles. These experiments were intended to show the effect of inclination on the rate of settling. What actually happens is that the height, L, through which the sediment falls in the upright cylinder is reduced to the height, $h = d \sec \theta$, as is shown in Fig. 7, when the cylinder is inclined at an angle, θ. A circulatory motion is steadily induced in the liquid, probably due to the downward slipping of the sediment on the side of the glass.

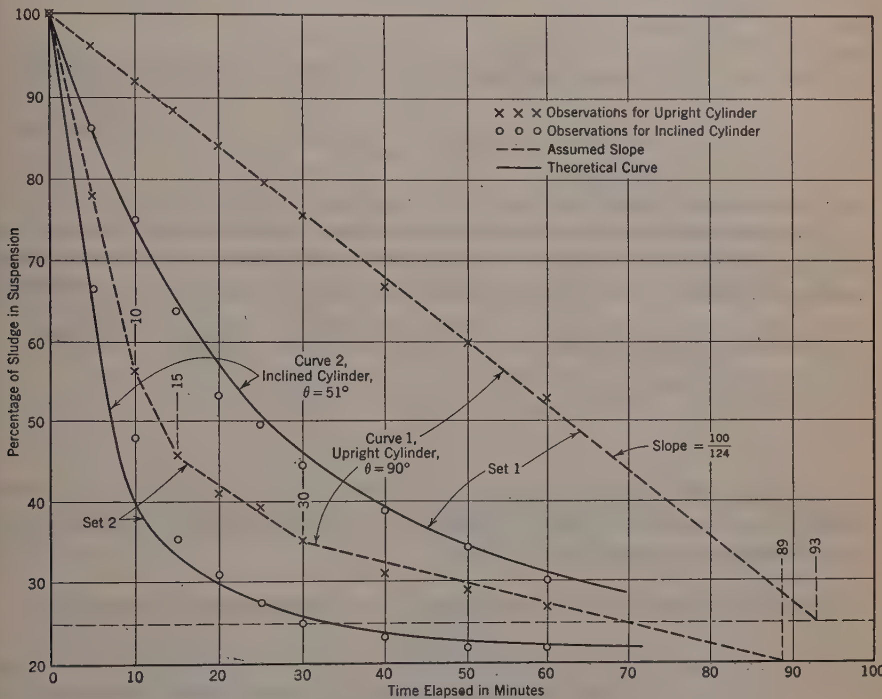

FIG. 8.

Consequently, there is: (1) Quiescent settling through the height, L, in the upright cylinder; and, then (2), turbulent settling through the height, $h = d \sec \theta$. Two sets of curves from the report by Messrs. Dilling and Pearse are shown in Fig. 8; in both sets, Curve (1) is for the upright cylinder ($\theta = 90°$), and Curve (2) is for the cylinder inclined at an angle of $\theta = 51$ degrees. Messrs. Dilling and Pearse do not give the dimensions of the cylinders. It will be assumed that $d = \frac{L}{6}$. From the description given of the circulation in the inclined cylinder, the turbulence may be assumed to be complete for the sludge of the experiments. If, from the curve for the upright cylinder, the distribution of hydraulic values of the sediment are determined, it should be possible to predict the behavior of the sediment in the inclined cylinder.

Curve (1), Set 1, for the upright cylinder, shows the sediment to have quite a uniform value, as far as the experiment has been run. The curves are not extended far enough to determine the quantity of colloidal and near-colloidal matter in the sediment, but all those for the inclined cylinders in this particular experiment seem to approach a horizontal line 25 units above the horizontal axis; for this reason the value of B_c was adopted. Since there is only one type of settleable material, $B_s = 100 - 25 = 75$. This value may also be determined by the use of Equation (18) (see Fig. 8, Set 1); thus, $B_s = \frac{93 \times 100}{124} = 75.$

The equation for the sedimentation curve in the quiescent state becomes:

$$B_a = 75\left(1 - \frac{a}{93}\right) + 25 \qquad (32)$$

in which the first term drops out after the time, $a = 93$.

To derive the formula of Curve (2) from Equation (32), it must be remembered that the time of subsidence has been reduced in the same ratio as the maximum height through which the particles fall; that is, the time of subsidence in this instance is $\frac{93}{(6 \cos 51°)} = \frac{93}{3.78} = \frac{1}{0.04065}$. The formula for Curve (2), Set 1, is,

$$B_a = 75\, e^{-0.04065a} + 25 \qquad (33)$$

Equation (33) is the formula for Curve (2), Set 1, in Fig. 8, and Table 1(*a*) shows some values computed from this equation compared with the corresponding experimental values.

TABLE 1.—COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES OF B_a (FIG. 8).

By:	(*a*) SET 1, FIG. 8, FOR VALUES OF *a*:					(*b*) SET 2, FIG. 8, FOR VALUES OF *a*:				
	0	10	20	40	60	0	10	20	40	60
Formula	100	75	58.3	39.8	31.5	100	39.6	29.9	23.7	21.5
Experiment	100	75	53	39	30	100	48	31	23	21.5

Curve (1), Set 2, Fig. 8, for the upright cylinder, represents sediment with a variety of hydraulic values. For reasons similar to the previous case, B_c is assumed as 20. As shown in Fig. 8, four slopes are taken, and their values are determined, by scaling, to be: $D_1 = 4.350$; $D_2 = 2.195$; $D_3 = 0.700$; and $D_4 = 0.224$, and the corresponding times of subsidence are (see Fig. 8, Set 2): $t_1 = 10$; $t_2 = 15$; $t_3 = 30$; and $t_4 = 89$.

From Equation (18): $B_1 = t_1\,(D_1 - D_2) = 10\,(4.350 - 2.195) = 21.55$; and, $B_2 = 22.50$; $B_3 = 14.30$; and $B_4 = 20$. If there were no errors in the process, these would be the actual values of B. The scaling has not been done very accurately, however, and the sum of these quantities (which, by

Equation (15), should be 80) is 78.35. Stating these quantities in round numbers: B = 22, 23, 15, and 20, respectively. Consequently, the equation of Curve (2), Set 2, is:

$$B_a = 22\, e^{-\frac{3.78a}{10}} + 23\, e^{-\frac{3.78a}{15}} + 15\, e^{-\frac{3.78a}{30}} + 20\, e^{-\frac{3.78a}{89}} + 20 \ldots\ldots (34)$$

In this case, as in that of Equation (33), and for the same reason, the times of subsidence must be multiplied by the factor, $\frac{1}{3.78}$. This curve is plotted in Fig. 8, Set. 2. Table 1(b) shows values computed from Equation (34) compared with corresponding experimental values.

The writer is not acquainted with the results of other experiments which would illustrate these relations in greater detail. He has seen many curves of observations on actual tanks, but none with the corresponding curves for quiescent settling, and no experiments at all from which an estimate might be made of the transition points from one state of turbulence to another. As has been stated, the distribution function cannot be found from the curve for turbulent sedimentation, so that wherever the writer has applied the general formula, Equation (31), it has been a matter of mere curve fitting.

The next example is selected at random from data made available by H. N. Lendall, M. Am. Soc. C. E. The small circles in Fig. 9 represent observations on the rate at which untreated sewage settles.

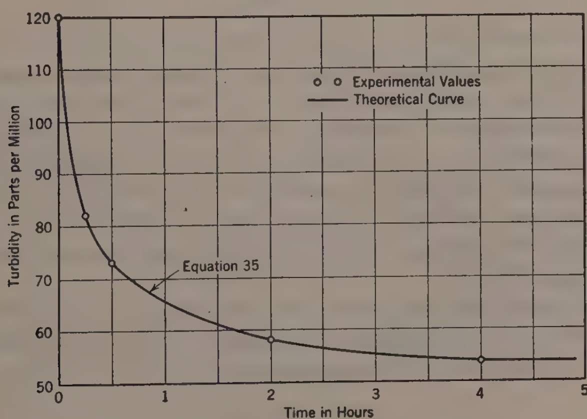

FIG. 9.

To fit a curve to these data by Equation (31) it was assumed that one term would be sufficient to take into account the portion of the sediment for which the turbulence in the basin was incomplete, and two terms for the portion for which the turbulence was complete; that is, Equation (31) became simply:

$$B_a = B_1 \left(1 - \frac{a}{k_1\, t_1}\right) + B_2\, e^{-\frac{a}{t_2}} + B_3\, e^{-\frac{a}{t_3}} + B_c \ldots\ldots\ldots (35)$$

To a certain extent the value one selects for the times of subsidence is a matter of choice. Fig. 1 shows the B-values determined from a certain distribution function for one choice of t-values, and Fig. 10 shows the B-values determined from the same distribution function for another choice of t-values. It was assumed that $k_1 t_1 = 15$ (since k_1 is not known, it is impossible to state the corresponding values, t_1); $t_2 = 30$; and $t_3 = 120$. By trial and error, or by any other method of curve fitting, the B-values are determined to be 25, 30, 13, and 52, respectively.

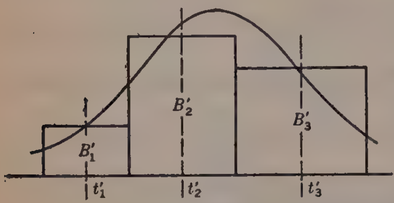

FIG. 10.

Equation (35) then, becomes,

$$B_a = 25\left(1 - \frac{a}{15}\right) + 30\, e^{-\frac{a}{30}} + 13\, e^{-\frac{a}{120}} + 52 \ldots\ldots\ldots\ldots (36)$$

in which it must be remembered that the first term drops out for values of a greater than 15.

In a way, of course, this curve-fitting procedure furnishes a means of determining the distribution function (the B-values), but since the distribution function is not what one usually wants, no importance should be attached to it.

CONCLUSIONS

Equations (18) and (31) are the most important products of this investigation. Through them one may predict how different kinds of sediment will settle under different tank conditions. Equation (18) gives the relation between the distribution of hydraulic properties of the sediment and its settling characteristics in a quiet tank. By means of this equation the distribution of hydraulic values of a sample of sludge may be determined by observing its settling behavior in a glass cylinder, for instance. Knowing this distribution, its settling characteristics in an actual tank may be predicted by Equation (31).

Before any practical application can be made of the theory herein developed, however, it is necessary to determine experimentally the constants, (t_i, t_c, k_r, etc.), suggested in this investigation.

APPENDIX

NOTATION

a = time during which sedimentation occurs; $p(a)$ = function of a;
b = quantity of solids, B, per unit volume;
c = a subscript denoting "colloidal", "complete", or "critical", as defined in each case;
d = inside diameter of a glass cylinder;

f = g = } functional symbols;

h = height; depth of a settling basin;

i = a subscript denoting "incomplete";

k = a coefficient; k_1, k_2, etc. = coefficients that produce the average times of subsidence, $k_1 t_1$, $k_2 t_2$, etc., under certain conditions of turbulence for particles with times of subsidence in a still basin equal to t_1, t_2, etc.;

m = a substitution factor $= -\frac{a}{n\,t}$; as a subscript denoting "minimum" when used with times of settling, and "maximum" when used with hydraulic values, $t_m = \frac{h}{v_m}$;

n = a number, such as a number of parts;

p = a functional symbol;

s = a number of parts, less than n; as a subscript, s denotes "settleable";

t = time of subsidence; t_m = minimum time corresponding to v_m; t_M = maximum time corresponding to v_M; t_1 and t_2 = times corresponding to v_1 and v_2, respectively; t_r = time corresponding to v_r; t_c = time required for complete or critical turbulence; $\phi(t)$ = distribution function of hydraulic values of suspended matter; as a subscript, t denotes "total";

v = a hydraulic value or the limiting velocity of a particle as it settles in a liquid; v_m = a maximum value; v_M = a minimum value; v_i = limit value for incomplete turbulence (turbulence is incomplete for $v_i < v$); v_c = limit value for complete or critical turbulence (turbulence is complete for $v_i < v < v_c$ and critical for $v < v_c$);

y = distance of a suspended particle above the floor of a settling basin;

z = quantity of solids; B_a = quantity remaining in suspension at

A = distance from the water surface to a given depth;

B = horizontal area of a tank or stilling well;

the end of time, a; B_t = total quantity in a given basin; B_s = quantity of settleable matter in a basin; B_c = quantity in a colloidal state, including that part of the settleable solids with hydraulic values, v, so small that, for practical purposes, they may be considered as in colloidal suspension; B_1, B_2, etc., = quantities with hydraulic values, v_1, v_2, etc.; B_r = quantity with an average hydraulic value, v_r;

D = a derivative of B_a in Equation (14);

F = a functional symbol;

L = length of a glass cylinder;

M = a subscript denoting "maximum", when used with t; "minimum" when used with v;

θ = inclination of a glass cylinder, with the horizontal;

ϕ = a function; $\phi(t)$ = distribution function of hydraulic values of suspended matter.

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PAPERS

WIND STRESSES IN REINFORCED CONCRETE ARCH BRIDGES

By A. A. Eremin[1], Assoc. M. Am. Soc. C. E.

Synopsis

Ordinarily, the effect of wind forces is slighted in the design of arch bridges despite the fact that wind stresses in these structures are often considerable. In the Ammer Arch Bridge,[2] near Echelsbach, Germany, with a span of 426 ft, the stresses due to wind constitute about 55% of the total, including live load, dead load, and temperature changes. On the other hand, the excessively heavy wind-bracing sometimes built into reinforced concrete arch bridges results in an uneconomic and unsightly design.

Introduction

The usual assumptions made in designing reinforced concrete rigid frames have been adopted in this paper. For example, the concrete was assumed to be elastic within the range of working stresses, from which it follows that strain is proportional to stress. The effect of plastic flow on the distribution of stress was not considered.

Notation.—The notation of this paper is defined where first introduced and summarized for reference, in the Appendix. A definition of the torsion factor, F, has been stated by J. Charles Rathbun, M. Am. Soc. C. E.[3] in the following form:

$$F = \frac{b^3\,t^3}{3.58\,(b^2 + t^2)} \qquad (1)$$

in which b is the width, and t is the thickness of the arch rib measured radially.

Arch Rib Without Bracing

Obviously, the single arch rib, fixed at the abutments and subjected to lateral forces, is statically indeterminate and requires six equations for its

Note.—Discussion on this paper will be closed in March, 1936, *Proceedings*.

[1] Assoc. Bridge Designing Engr., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[2] *Beton und Eisen*, March, 1930.

[3] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 38, Equation (28).

solution. In the case of a symmetrical structure, however, the computation of wind stresses is simplified, and, furthermore, the effects of shear and direct stresses in the rib may be neglected without introducing serious error.

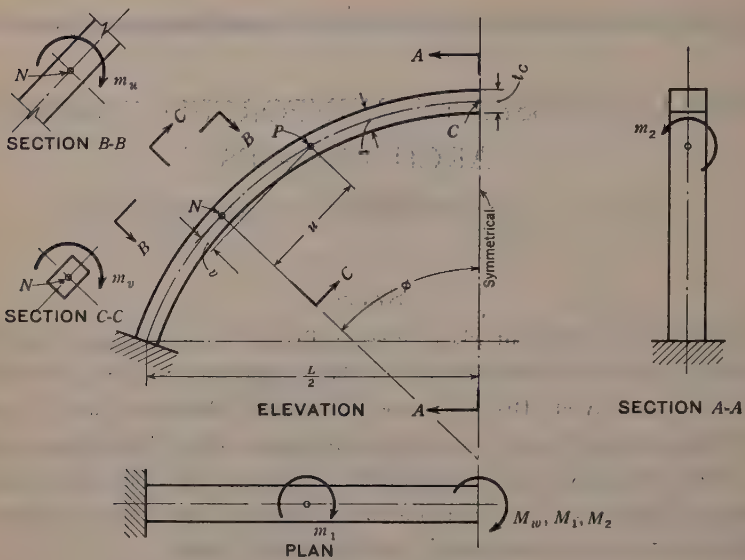

Fig. 1.—Symmetrical Arch Rib.

If the temperature of the arch is assumed constant the internal work due to elastic formation is expressed as,

$$W = \frac{1}{2}\int\left(\frac{m_u^2}{EI} + \frac{m_v^2}{GF}\right) ds \qquad (2)$$

Applying Castigliano's theorem, the virtual displacement at some point, *P* (Fig. 1), may be derived from Equation (2) as follows:

$$\theta = \int (m_u\, C_{1m} + \gamma\, m_v\, C_{1t})\, dw \qquad (3)$$

and,

$$\tau = \int (m_u\, C_{2m} + \gamma\, m_v\, C_{2t})\, dw \qquad (4)$$

in which $\frac{ds}{EI} = dw$ and C_{1m} and C_{1t} are, respectively, the partial differential coefficients representing the moment and torsion produced by a unit moment, $m_1 = 1$, applied at Point *P*, and C_{2m} and C_{2t} are, respectively the partial differential coefficients representing the moment and torsion produced by a unit moment, $m_2 = 1$ at Point *P*.

Professor E. Mörsch has shown[4] that, when a symmetrical arch rib is exposed to symmetrical lateral wind forces, such as an ideal uniform wind,

[4] *Beton und Eisen*, 1923, p. 53.

the only bending moment that occurs at the crown is M_w about the vertical axis. It is obvious that the torsion and shear stresses at this point are equal to zero, as are the bending moments about the horizontal axis, since the direct stresses are assumed to be negligible. Consequently, for the structure in Fig. 1, which is fixed at the springing, the bending and torsion moments produced by wind are expressed as follows:

$$m_u = - \int w\, t\, u\, ds + M_w \cos \phi \quad \text{.........(5)}$$

and,

$$m_v = + \int w\, t\, v\, ds - M_w \sin \phi \quad \text{.........(6)}$$

The partial differential coefficients of the unit moments, $m_1 = 1$ and $m_1 = 1$, are:

$$C_{1m} = + \cos \phi \quad \text{.........(7)}$$

$$C_{1t} = - \sin \phi \quad \text{.........(8)}$$

$$C_{2m} = + \sin \phi \quad \text{.........(9)}$$

and,

$$C_{2t} = + \cos \phi \quad \text{.........(10)}$$

Therefore, the bending deformation due to wind may be written, from Equations (4) to (10):

$$\theta_w = - \int \left(\cos \phi \int w\, t\, u\, ds + \gamma \sin \phi \int w\, t\, v\, ds\right) dw + M_w \int (\cos^2 \phi + \gamma \sin^2 \phi)\, dw \quad \text{.........(11)}$$

and the torsion deformation will be:

$$\tau_w = - \int \left(\sin \phi \int w\, t\, u\, ds - \gamma \cos \phi \int w\, t\, v\, ds\right) dw + M_w \int (1 - \gamma) \sin \phi \cos \phi\, dw \quad \text{.........(12)}$$

The tangent at the crown does not move when a symmetrical arch rib is subjected to uniformly distributed lateral forces. Therefore, by equating the bending deformation at the crown to zero, M_w may be expressed:

$$M_w = \frac{\int \left(\cos \phi \int w\, t\, u\, ds + \gamma \sin \phi \int w\, t\, v\, ds\right) dw}{\int (\cos^2 \phi + \gamma \sin^2 \phi)\, dw} \quad \text{.......(13)}$$

In a similar manner the equations for bending and torsion deformations and the moment at the crown produced by the symmetrical moments, $m_1 = 1$, may be written:

$$\theta_1 = \int (\cos^2\phi + \gamma \sin^2\phi)\, dw + M_1 \int (\cos^2\phi + \gamma \sin^2\phi)\, dw \quad (14)$$

$$\tau = \int (1 - \gamma) \sin\phi \cos\phi\, dw + M_1 \int (1 - \gamma) \sin\phi \cos\phi\, dw \quad (15)$$

and,

$$M_1 = -\frac{\int (\cos^2\phi + \gamma \sin^2\phi)\, dw}{\int (\cos^2\phi + \gamma \sin^2\phi)\, dw} \quad (16)$$

In a similar manner the equations for bending and torsion deformations, and moment at the crown produced by the symmetrical moments, $m_2 = 1$, are:

$$\theta_2 = \int (1 - \gamma) \sin\phi \cos\phi\, dw + M_2 \int (\cos^2\phi + \gamma \sin^2\phi)\, dw \quad (17)$$

$$\tau_2 = \int (\sin^2\phi + \gamma \cos^2\phi)\, dw + M_2 \int (1 - \gamma) \sin\phi \cos\phi\, dw \quad (18)$$

and,

$$M_2 = \frac{\int (\gamma - 1) \sin\phi \cos\phi\, dw}{\int (\cos^2\phi + \gamma \sin^2\phi)\, dw} \quad (19)$$

The numerators of Equations (16) and (19) must be integrated from the points of application of the unit moments, m_1 and m_2, to the springing and the denominator from the crown to the springing section.

Braced Arch Ribs

Reinforced concrete arch bridges are generally constructed with lateral braces between the ribs. Each of these braces adds six more unknowns to the system of forces in the rib. When the structure is symmetrical, however, the number of unknowns is reduced; but in order to derive simple expressions for stresses in the bracing, certain assumptions are necessary. For example, it is assumed that each rib carries the same wind load. Furthermore, the intersections of the braces with the rib are assumed to be fixed in space and, therefore, the points of contraflexure in the braces will be at the mid-point in each case, and will also be fixed in space.

One-half of one arch rib is analyzed by substituting for the remainder of the structure the forces, R_1 and R_2, as shown in Fig. 2. Direct stresses in the members are neglected. Forces R_1 and R_2, therefore, will keep the mid-point of the brace fixed in position and the equations for these reactions can be developed by an application of Maxwell's theorem.

Two increments of displacement in the arch rib and bracing are considered: First, that produced by wind pressures on the half arch rib in Fig. 2, without the reactions, R_1 and R_2; and, second, that produced by the forces, R_1 and R_2, in order to translate the mid-point of the cross-brace back to its original position.

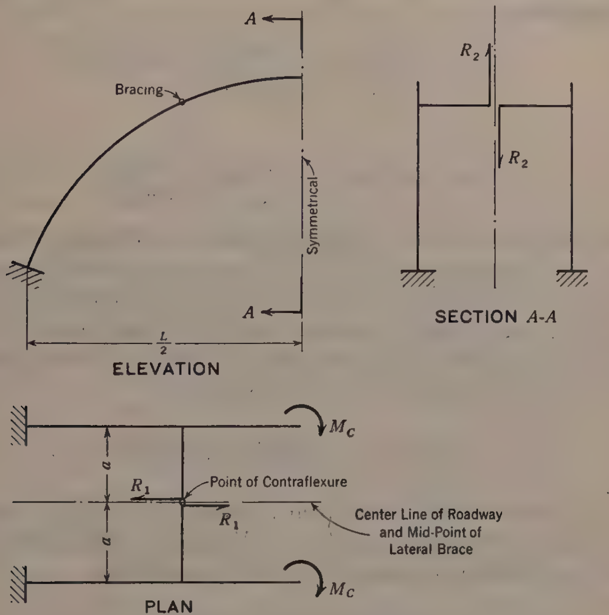

FIG. 2.—SYMMETRICAL ARCH RIBS, WITH LATERAL BRACE.

In Fig. 2, with the forces, R_1 and R_2, removed, the displacement in the direction of the force, R_1, is $\tau_w a$, and in the direction of R_2, $\theta_w a$. If, instead of the uniform lateral wind load, the half arch in Fig. 2 were loaded with the force, R_1, the deflections would be, respectively, in the direction of R_1 $\left(\text{since the deflection of a cantilever beam is } \frac{R_1 a^3}{3 E I_b}\right)$:

$$\Delta = R_1 a \left(\tau_1 a + \frac{a^2}{3 E I_b}\right) \text{.....................(20)}$$

and, in the direction of R_2;

$$\Delta = R_1 a \theta_1 a \text{.........................(21)}$$

in which the products, $R_1 a$ and $R_2 a$, are moments of the forces, R_1 and R_2, about the intersection of the cross-brace and the arch rib.

If, instead of the reaction, R_1, the arch is loaded with the reaction, R_2, the corresponding displacements are:

In the direction of R_1:

$$\Delta = R_2 a \tau_2 a \text{.........................(22)}$$

and, in the direction of R_2:

$$\Delta = R_2 a \left(\theta_2 a + \frac{a^2}{3 E I_a} \right) \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Consequently, according to Maxwell's theorem, the displacements may be expressed as follows:

$$\tau_w = R_1 \left(a \tau_1 + \frac{a^2}{3 E I_b} \right) + R_2 a \tau_2 \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

and,

$$\theta_w = R_1 a \theta_1 + R_2 \left(a \theta_2 + \frac{a^2}{3 E I_a} \right) \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Equations (24) and (25) were simplified by dividing through by the length, a. The deformations, τ and θ, are computed by means of Equations (11) to (18), inclusive, and with these values substituted in Equations (24) and (25), simultaneous solutions will yield the values of R_1 and R_2. Removing the right half of the arch has the effect of substituting a moment at the crown of the remaining left half equal to,

$$M_c = M_w + M_1 R_1 a + M_2 R_2 a \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

The bending moment and torsion at any other section of the rib can be computed by statics. Formulas similar to Equations (24) and (25) may be written for any number of braces between the arch ribs.

Illustrative Example

In order to illustrate the application of the foregoing equations, the wind load stresses in the arch shown in Fig. 3 will be computed. The ribs are fixed at the springing and braced at a distance of 26.28 ft measured hori-

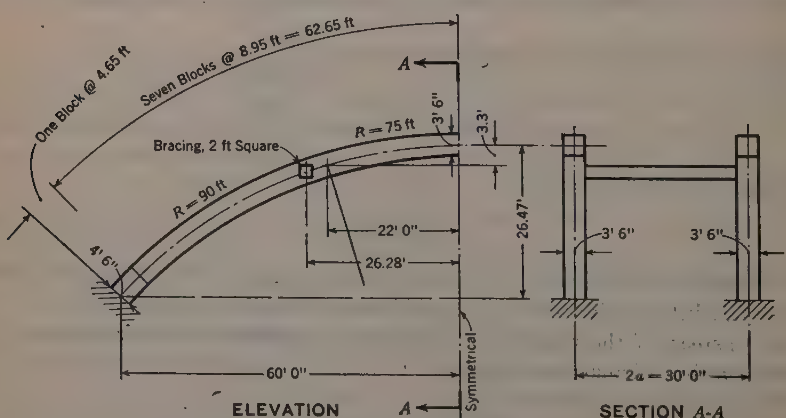

Fig. 3.—Illustrative Example.

zontally from the crown. The theoretical span of the arch is 120 ft and the thickness of the arch rib varies according to the formula,

$$t = t_c\,(1 + 0.446\,\phi^2) \qquad \text{(27)}$$

in which t_c equals the thickness of the rib at the crown and ϕ equals the angle between the radial line and the vertical through the crown. Since the lateral cross-brace is 2 by 2 ft in section, $I_a = I_b = 1.33$ ft^4, and,

$$\frac{a^2}{3\,E\,I_a} = \frac{a^2}{3\,E\,I_b} = \frac{56.26}{E} \qquad \text{(28)}$$

The steel reinforcement is not taken into consideration in computing the moment of inertia. Such precision would be useless because the theory of wind load distribution on exposed surfaces is still so inadequate that extreme refinements in other factors are inconsistent.

Although the complete computations for this numerical example are not recorded herein, it is believed that they are given in sufficient detail for the guidance of those who are familiar with the design of arch bridges.

A wind load of $w = 45$ lb per sq ft is assumed to act horizontally and at right angles to the plane of the arch rib. In order to integrate the elastic and geometric properties, the rib was divided into eight blocks as indicated in Fig. 3, the values being given in Table 1. The modulus of elasticity of concrete in compression was assumed equal to 2.5 G, the modulus of elasticity of concrete in shear.

TABLE 1.—VALUES OF INTEGRATIONS IN TERMS OF $\dfrac{1}{E}$

INTEGRALS	For half arch rib	From springing to bracing
$\int (\gamma - 1) \sin\phi \cos\phi \dfrac{dw}{2}$	0.543	0.374
$\int (\sin^2\phi + \gamma \cos\phi)\, dw$	6.542	3.461
$\int (\cos^2\phi + \gamma \sin^2\phi)\, dw$	5.199	3.043
$\int \left(\cos\phi \int w\,t\,u\,ds + \gamma \sin\phi \int w\,t\,v\,ds\right) dw$	516 660	481 360
$\int \left(\sin\phi \int w\,t\,u\,ds + \gamma \cos\phi \int w\,t\,v\,ds\right) dw$	....	246 720

Selecting values from Table 1, Equations (13), (14), and (19) may be solved to determine the crown moments, M_w, due to the wind load and those due to the symmetrical moments, M_1 and M_2, thus: $M_w = \dfrac{516\,660}{5.199} = 99\,377$ ft-lb; $M_1 = -\dfrac{3.043}{5.199} = -0.585$ ft-lb; and, $M_2 = \dfrac{0.374}{5.199} = 0.072$ ft-lb.

The angular deformations $\left(\text{multiplied by } \frac{1}{E}\right)$ are computed by means of Equations (11) to (18), as follows:

$$\theta_w = -\ 481\,360 + 99\,377 \times 3.043 = -\ 178\,956$$

$$\tau_w = -\ 246\,720 + 99\,377 \times 0.374 = -\ 209\,652$$

$$\theta_1 = +\quad 3.043 - \quad 0.585 \times 3.043 = +\ 1.263$$

$$\tau_1 = -\quad 0.374 + \quad 0.585 \times 0.374 = -\ 0.155$$

$$\theta_2 = -\quad 0.374 + \quad 0.072 \times 3.043 = -\ 0.155$$

and,

$$\tau_2 = +\quad 3.461 - \quad 0.072 \times 0.374 = +\ 3.434$$

It follows from Maxwell's theorem, furthermore, that $\tau_1 = \theta_2$. The bending moment and the torque were considered positive in the direction indicated in Fig. 1. The same is true of the angular deformations. The forces in the lateral cross-brace may be derived from Equations (24) and (25) by substituting the necessary angular deformations; thus:

$$(56.25 - 0.16 \times 15)\ R_1 + 3.434 \times 15\ R_2 = 209\,652 \ldots\ldots (29)$$

and,

$$1.263 \times 15\ R_1 + (56.26 - 0.16 \times 15)\ R_2 = 178\,956 \ldots\ldots\ldots (30)$$

which, solved simultaneously, yield $R_1 = 1.074$ lb and $R_2 = 2.946$ lb.

The bending moment about the vertical axis at the crown is computed by Equation (26), thus: $M_C = 99.377 - 0.585 \times 1.074 \times 15 + 0.072 \times 2\,946 \times 15 = 92\,785$ ft-lb. Therefore, the maximum unit compressive stress in the arch rib at the crown, produced by a wind-load bending moment, M_C, will be, $f_c = \frac{M}{S} = \frac{6 \times 92\,785 \times 12}{42 \times 42 \times 42} = 90.5$ lb per sq in., which is about 9% of the maximum compressive working stress (1 000 lb per sq in.) generally used in the design of reinforced concrete arch bridges.

The maximum wind-load bending moment at the end of the lateral cross-brace is $M = R_2 A = 2\,946 \times 15 = 44\,190$ ft-lb, which is about 126% of the dead load bending moment in the brace at that point. For comparison, the dead load bending moment is $M_d = \frac{D\,(2\,a - b)}{12} = \frac{600 \times 26.5 \times 26.5}{12} = 35\,113$ ft-lb.

When this problem occurs in connection with a through bridge, hangers may be assumed flexible enough so that their influence on the rigidity of the rib may be neglected. If the roadway is poured monolithic with the abutment, it will transmit its own wind pressure to the abutment without materially affecting stresses in the arch ribs and the braces. Therefore, only the wind pressure on the side of the arch itself needs to be assumed to be carried by the ribs and the bracing.

The foregoing development may lead some readers to criticize the paper for lack of precision. However that may be, the illustrative example is offered merely for the purpose of demonstrating the application of the formulas developed in this paper. The relative accuracy can be controlled by the designer himself.

CONCLUSION

Although the wind stresses computed in this paper have not been verified by tests, the formulas were developed from the well-known elastic equations. Furthermore, the method of computing the torsion factor and the influence of torque on the deformation of an arch have been successfully proved by tests of skew arches reported in 1928 by Professor Rathbun.[5]

The effects of lateral wind forces on the forces in the plane of the arch rib are so slight as to be negligible and, therefore, the lateral forces may be treated separately.

In the case of arches subjected to unsymmetrical lateral forces, the equations in this paper may be modified as the particular case demands. Forces inclined to the plane of the arch rib may be resolved into their components. Furthermore, the eccentric forces applied to the arch through the roadway deck may be divided into components of force acting in the plane of the arch rib and a torque equal to the vertical force times eccentricity. In this case, the torque may also be treated as a moment produced by horizontal lateral forces.

It is important to study the practical application of the elastic equation to problems in which forces in space occur. Such study may eliminate many obsolete assumptions in time, thus resulting in further economic and æsthetic improvements in the design of such structures.

APPENDIX

NOTATION

The following notation is adopted for use in this paper:

a = one-half the length of a lateral csoss-brace, measured from its intersection with the axis of the arch rib; as a subscript, a denotes "vertical axis";

b = breadth; width of arch rib; as a subscript, b denotes "horizontal axis";

c = a subscript denoting "concrete";

d = a subscript denoting "dead load";

f = unit stress; f_c = unit compressive stress in concrete;

m = moment component; m_u = a component of the moment acting in a tangential plane through Point N (Fig. 1); m_v = a component of the moment acting in a radial plane through Point N.

ds = length of an element of the arch rib = $E\ I\ dw$;

t = thickness of the arch barrel, measured radially;

[5] *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 135.

u = a co-ordinate of any point, P, measured at right angles to a radial line through Point N; as a subscript, u refers to a plane perpendicular to a radial line through Point N;

v = a co-ordinate of any point, P, referred to a Point N, measured parallel to the radial line through Point N (see Fig. 1); as a subscript, v refers to a radial plane;

w = uniformly distributed, lateral, wind load; $dw = \frac{ds}{EI}$; as a subscript, w denotes "due to wind load";

$\gamma = \frac{EI}{GF}$; or, if $E = 2.5\ G$, $\gamma = \frac{2.5I}{F}$;

C = a partial differential coefficient; C_{1m} and C_{2m} = bending moment coefficients corresponding to $m_1 = 1$ and $m_2 = 1$ applied at Point P; C_{1t} and C_{2t} = torque coefficients corresponding to moments, $m_1 = 1$ and $m_2 = 1$, applied at Point P; as a subscript, C denotes "at the crown";

D = total dead load;

E = modulus of elasticity of concrete in tension and compression;

F = a torsion factor;

G = modulus of elasticity of concrete in shear;

I = moment of inertia of the cross-section area of the arch cut by a radial plane; I_a = moment of inertia of a section of a cross-brace, about a vertical axis through the center of gravity; I_b = moment of inertia of a section of bracing about a horizontal axis through the center of gravity;

L = span length;

M = bending moment; M_C = bending moment at the crown; M_w = bending moment due to wind load; M_1 = bending moment at the crown due to the symmetrical moments, $m_1 = 1$; M_2 = bending moment at the crown due to the symmetrical moments, $m_2 = 1$; M_d = dead load bending moment;

R = reaction; R_1 = horizontal force applied to the center of a wind-brace; R_2 = vertical force applied to the center of a wind-brace;

S = section modulus;

T = torsion; T_1 = torque corresponding to a unit moment, $m_1 = 1$, applied at Point P;

W = internal work due to the elastic deformation of the arch rib;

Δ = displacement;

θ = bending deformation at Point N, about the vertical axis; θ_w = bending deformation produced by the wind; θ_1 = bending deformation produced by $m_1 = 1$; θ_2 = bending deformation produced by $m_2 = 1$;

τ = torsion deformation at Point N about the horizontal axis; τ_w = deformation produced by wind; τ_1 = deformation produced by $m_1 = 1$; τ_2 = deformation produced by a moment, $m_2 = 1$;

ϕ = central angle locating any point, N, on the arch axis, with relation to a radius through the crown.

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PAPERS

SUCCESSIVE ELIMINATION OF UNKNOWNS IN THE SLOPE DEFLECTION METHOD

By John B. Wilbur[1], Assoc. M. Am. Soc. C. E.

Synopsis

A method of applying the usual slope deflection equations[2] is presented in this paper, which by successively expressing all the unknown elements in terms of a few unknowns makes an exact solution possible without the formal solution of a large number of simultaneous equations.

The method is described by showing its application to three types of structures: Continuous beams, building frames with shallow wind-bracing, and the Vierendeel truss. Certain principles are then enumerated, which may be extended to other structures that may be solved by the slope deflection method.

Notation

Throughout this paper, it will be assumed that the reader is familiar with the slope deflection theory. The usual slope deflection notation will be used, as follows:

d = relative deflection measured normal to the axis of a member, between the ends of the member;
h = height of a column, or distance between floors; story height;
p = panel length;
C = fixed-end moment at Point B on Beam BC, due to any load, P; a resisting moment;
E = modulus of elasticity in tension and compression;
I = moment of inertia;
K = ratio of the moment of inertia, to the length of a structural member;
M = moment of an external couple; M_{AB} = moment at End A, of Beam AB, etc. (see, also, C);

Note.—Discussion on this paper will be closed in March, 1936, *Proceedings*.

[1] Asst. Prof., Civ. Eng., Mass. Inst. Tech., Cambridge, Mass.

[2] "Analysis of Statically Indeterminate Structures by the Slope Deflection Method", by W. M. Wilson, F. E. Richart, and Camillo Weiss, Members, Am. Soc. C. E., *Bulletin 108*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

P = a concentrated load;

R = ratio of the increment in horizontal deflection occurring in a story, to the story height, the floors being identified by subscripts;

θ = a change in the slope of the tangent to the elastic curve of a member.

Continuous Beams

For a slope deflection solution of the structure shown in Fig. 1(*a*), there are five unknowns: θ_A, θ_B, θ_C, θ_D, and θ_E. For the order of using the slope deflection equations to be outlined, θ_A will be termed the permanent unknown, since all the other values of θ will be expressed, successively, in terms of θ_A, which will then be evaluated. The slope deflections θ, at the remaining supports will be termed the temporary unknowns.

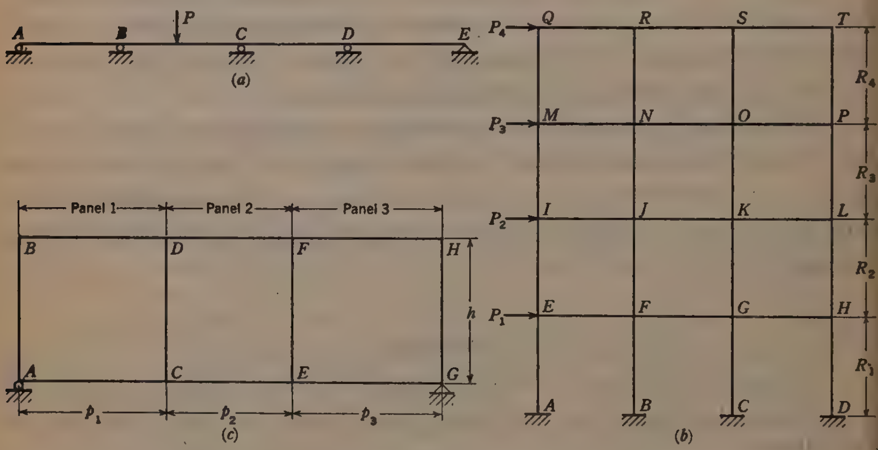

Fig. 1.

At either end support of a continuous beam, a known element exists; either the moment or the slope is zero, or the moment is known, as when the beam is a cantilever beyond the support. In Fig. 1(*a*) the moment at each end is zero. Thus, taking θ_A as the permanent unknown:

(1) Express θ_B in terms of θ_A by writing,

$$M_{AB} = 2\ E\ K_{AB}\ (2\ \theta_A + \theta_B) = 0 \qquad (1)$$

or,

$$\theta_B = -\ 2\ \theta_A \qquad (2)$$

(2) Express θ_C in terms of θ_A by writing $M_{BA} + M_{BC} = 0$, in which,

$$M_{BA} = 2\ E\ K_{AB}\ (2\ \theta_B + \theta_A) = 2\ E\ K_{AB}\ (-\ 3\ \theta_A) \qquad (3)$$

and,

$$M_{BC} = 2\ E\ K_{BC}\ (2\ \theta_B + \theta_C) \pm C = 2\ E\ K_{BC}\ (-4\ \theta_A + \theta_C) \pm C. \quad (4)$$

in which C is the fixed-end moment at Point B on Beam BC due to any load, P.

(3) Similarly, by $M_{CB} + M_{CD} = 0$ (that is, $\Sigma M = 0$ at Support C), express θ_D in terms of θ_A.

(4) Similarly, by $\Sigma M = 0$ at Support D, express θ_E in terms of θ_A.

(5) The fact that $M_{ED} = 0$ enables a solution for θ_A since $M_{ED} = E\ K_{DE}\ (2\ \theta_E + \theta_D) = 0$, in which θ_E and θ_D are functions of θ_A.

(6) With θ_A known, and all other values of θ in terms of θ_A, the moments in the beam are easily determined by the slope deflection equation.

Thus, the continuous beam shown in Fig. 1(a) is analyzed without the formal solution of simultaneous equations. This fact is not dependent upon the number of spans. Had the beam been fixed at Point A, θ_B would have been taken as the permanent unknown, and the first step would have been to express θ_C in terms of θ_B by $\Sigma M = 0$ at Support B.

Building Frames with Shallow Wind-Braces

For a slope deflection solution of the structure shown in Fig. 1(b), there are twenty unknowns, namely, θ_E to θ_T, inclusive, and R_1, R_2, R_3, and R_4, which are the ratios of increment in horizontal deflection occurring in a story, to the story height, in the first, second, third, and fourth stories, respectively. For the proposed method, θ_E, θ_F, θ_G, and θ_H, are taken as the permanent unknowns. If the structure were symmetrical only θ_E and θ_F would be so taken. The procedure, using the slope deflection equations, is, as follows:

(1) Express M_{AE}, M_{EA}, M_{BF}, M_{FB}, M_{CG}, M_{GC}, M_{DH}, and M_{HD} in terms of θ_E, θ_F, O_G, θ_H, and R_1;

(2) Using the shear-story equation for the first story (that is, shear in the first story, times the first story height, $+ M_{AE} + M_{EA} + M_{BF} + M_{FB} + M_{CG} + M_{GC} + M_{DH} + M_{HD} = 0$), express R_1 in terms of θ_E, θ_F, θ_G, and θ_H;

(3) At Joint E, write $\Sigma M = 0$, where all moments may now be written in terms of θ_E, θ_F, θ_G, θ_H, θ_I, and R_2; from this equation express θ_I in terms of θ_E, θ_F, θ_G, θ_H, and R_2. Similarly, working from $\Sigma M = 0$ at Joints F, G, and H express θ_J, θ_K, and θ_L, respectively, in terms of θ_E, θ_F, θ_G, θ_H, and R_2;

(4) The moments in the ends of the second-story columns may now be written in terms of θ_E, θ_F, θ_G, θ_H, and R_2, and the application of the shear-story equation to the second story, enables the designer to express R_2, and, hence, θ_I, θ_J, θ_K, and θ_L, in terms of the permanent unknowns;

(5) Working progressively up the building in this manner, express all θ-values and all R-values in terms of the permanent unknowns; four equations, one for each unknown will still be available, namely, $\Sigma M = 0$ at Joints Q, R, S, and T. The formal solution of these four simultaneous equations yields values of θ_E, θ_F, θ_G, and θ_H; and,

(6) With all the R-values and all the temporary θ-values previously expressed in terms of the permanent unknowns, the moments in the structure are easily determined by the slope deflection equation.

Thus, the building frame shown in Fig. 1(b) is solved with the formal solution of only four simultaneous equations, one equation for each row of columns. This fact is independent of the number of stories. Symmetry would further reduce the labor since for the structure shown in Fig. 1(b), $\theta_E = \theta_H$ and $\theta_F = \theta_G$. Thus, a twenty-story, four-column, symmetrical bent[3] could be analyzed with the formal solution of only two simultaneous equations.

Illustrative Problem

To demonstrate the application of the proposed method to the case of building frames with shallow wind-braces, the structure shown in Fig. 2 may be solved, as follows: The shear-story equation applied to the first floor yields:

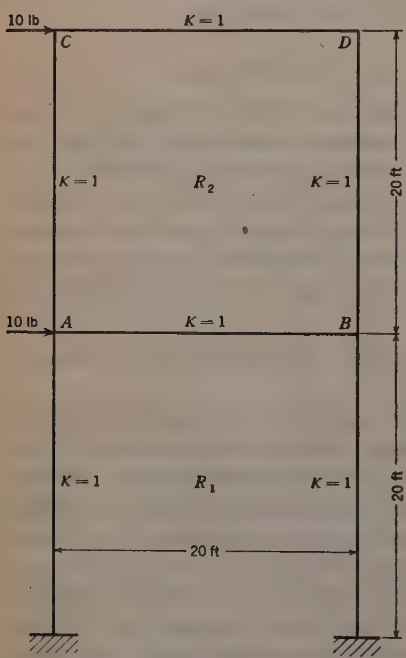

Fig. 2.

$$2\,[2\,E\,(2\,\theta_A - 3\,R_1) + 2\,E\,(\theta_A - 3\,R_1)] + 20\,(20) = 0$$

or,

$$3\,R_1 = 1.5\,\theta_A + \frac{100}{2\,E}$$

For the condition, $\Sigma M = 0$ at Joint A:

$$2\,\theta_A - 1.5\,\theta_A - \frac{100}{2\,E} + 3\,\theta_A + 2\,\theta_A + \theta_C - 3\,R_2 = 0$$

or,

$$\theta_C = 3\,R_2 + \frac{100}{2\,E} - 5.5\,\theta_A$$

Applied to the second floor, the shear-story equation yields:

$$2\left[\,2\,E\,(3\,\theta_A + 9\,R_2 + \frac{300}{2\,E} - 16.5\,\theta_A - 6\,R_2)\right] + 10 \times 20 = 0$$

or,

$$3\,R_2 = 13.5\,\theta_A - \frac{400}{2\,E};\ \text{and, } \theta_C = 8\,\theta_A - \frac{300}{2\,E}$$

[3] For example, see "Wind Stresses in the Steel Frames of Office Buildings", by W. M. Wilson and G. A. Maney, Members, Am. Soc. C. E., *Bulletin No. 80*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill., 1915.

Similarly, for the condition, $\Sigma M = 0$ at Joint C:

$$2\left(8\,\theta_A - \frac{300}{2\,E}\right) + \theta_A - 13.5\,\theta_A + \frac{400}{2\,E} + 3\left(8\,\theta_A - \frac{300}{2\,E}\right) = 0$$

and $\theta_A = \dfrac{40}{2\,E}$. Therefore, the deflection ratio, R_1, is expressed by $3\,R_1 = \dfrac{160}{2\,E}$; and the moment at the column base is $M = 2\,E\left(\dfrac{40}{2\,E} - \dfrac{160}{2\,E}\right) = -\,120$ ft-lb, the minus sign denoting that the couple, M, acts contra-clockwise on the column.

Vierendeel Truss

For the structure shown in Fig. 1(*c*), let $R = \dfrac{d}{h}$, in which d is the relative horizontal deflection between the top and bottom of any of the vertical members of the truss. Let R_1, R_2, and R_3 equal $\dfrac{d_1}{p_1}$, $\dfrac{d_2}{p_2}$, and $\dfrac{d_3}{p_3}$, respectively, in which d_1, d_2, and d_3 are the relative vertical deflections of the ends of the chord members in the first, second, and third panels, respectively. For the usual slope deflection solution of this structure, there would be twelve unknowns namely, θ_A to θ_H, inclusive, and R, R_1, R_2, and R_3. For the proposed method, θ_A, θ_B, and R are taken as the permanent unknowns, and the procedure is as follows:

(1) From $\Sigma M = 0$ at Joints A and B, express θ_C and θ_D, respectively, as a function of θ_A, θ_B, R, and R_1;

(2) Since the shear in Panel 1, times p_1 plus $M_{AC} + M_{CA} + M_{BD} + M_{DB}$, equals zero (the shear-panel equation), and all these moments may be written in terms of θ_A θ_B, R, and R_1, it follows that R_1 and, thus, θ_C and θ_D may be expressed in terms of θ_A, θ_B, and R;

(3) Similarly, by $\Sigma M = 0$ at Joints C and D, and the shear-panel equation for Panel 2, θ_E, θ_F, and R_2 may be expressed in terms of θ_A, θ_B, and R;

(4) Similarly, by $\Sigma M = 0$ at Joints E and F, and the shear-panel equation for Panel 3, θ_G, θ_H, and R_3 may be expressed in terms of θ_A, θ_B, and R;

(5) The three permanent unknowns may now be evaluated from the following simultaneous equations: $\Sigma M = 0$ at Joint G; $\Sigma M = 0$ at Joint H; and the shear-story equation applied to the vertical members of the bridge as a whole; and,

(6) With θ_A, θ_B, and R known, and the temporary unknowns already expressed in terms of these permanent unknowns, the moments are easily evaluated by the slope deflection equation.

Remarks

The foregoing examples show that it is necessary to take as permanent unknowns only one θ-value for each row of members parallel to the direction in which the solution is to progress across a structure, plus one R-value

for each panel between such rows (if there is more than one row), whenever the construction is such that these R-values do not equal zero. For the formal solution of simultaneous equations, there will be as many equations as there are permanent unknowns. It should be evident that solution in this manner does not actually avoid the solution of simultaneous equations, but, rather, provides a means of eliminating the unknowns progressively. The usual slope deflection equations could be written simultaneously, and if solved in the order described in this paper, would lead to the same solution. It seems to the writer, however, that there is little likelihood of this being done without purposely following the line of reasoning herein outlined.

A portion of the underlying research which forms the basis of this paper was submitted to Massachusetts Institute of Technology by the writer in 1933 in partial fulfillment of the requirements for the degree of Doctor of Science. Other phases of the broader subject have been published elsewhere under the title, "A New Method for Analyzing Stresses Due to Lateral Forces in Building Frames[4]", and "Distribution of Wind Loads to the Bents of a Building[5]."

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[4] *Journal*, Boston Soc. of Civ. Engrs., January, 1934, p. 45.
[5] *Loc. cit.*, October, 1935.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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REINFORCED CONCRETE MEMBERS UNDER DIRECT TENSION AND BENDING

By D. B. Gumensky[1], Assoc. M. Am. Soc. C. E.

Synopsis

The combination of compression and bending stresses is a common occurrence in design and has been very well treated in standard books on reinforced concrete. However, the combination of direct tension and bending is a problem that has received less attention from authors of engineering texts. This problem is met with in the design of Vierendeel trusses or particularly in the design of closed water conduits under pressure, where the conduit barrel is horizontal and acts as a complete elastic ring. The following presentation is an attempt to analyze the stress distribution in reinforced concrete members subjected to a combined stress due to direct tension and bending, and to suggest an easy method of solving these problems.

Assumptions

The assumptions used in the derivation of the formulas are the same as those generally accepted for the derivation of working formulas for reinforced concrete. They are as follows: (1) A straight-line distribution of stress; (2) concrete is an elastic substance; (3) tension is resisted entirely by steel; (4) bond between concrete and steel is perfect; (5) there are no initial stresses in concrete or steel; (6) the symbols denote only a numerical value of the quantities they represent; (7) vertical distances measured downward from the axis of a member are negative and upward distances are positive; (8) forces acting from right to left are positive and forces acting from left to right are negative; and (9) moments of forces about a point acting in a counter-clockwise direction are positive.

Notation.—The mathematical symbols are defined where they first appear in the paper and are summarized, for reference, in the Appendix.

Note.—Discussion on this paper will be closed in March, 1936, *Proceedings*.

[1] With Metropolitan Water Dist. of Southern California, Los Angeles, Calif.

Analysis and Solution Related to the Geometrical Axis of the Cross-Section of the Member

In analyzing indeterminate structures, the resultant moment, M, the direct pull, T, and the eccentricity of direct stress, $e\left(=\frac{M}{T}\right)$, at any one point, are usually related to the geometrical axis of the elastic ring or frame. This relation is maintained in the following presentation. It is generally accepted that the combination of an axial load and a couple will produce the same stresses as an equivalent eccentrically applied pull. In this paper, the latter will be used for the derivation of formulas. There are two distinct cases of stress distribution: (1) That in which the entire section is under tension; and (2) that in which part of the section is under compression.

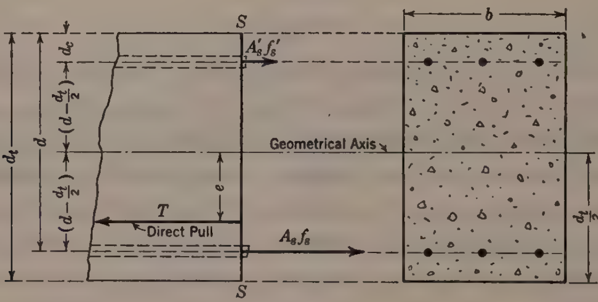

FIG. 1.

Case 1.—The Entire Section Under Tension.—This case has been excellently, but briefly, treated by Messrs. Taylor, Thompson, and Smulski.[2] Section S-S, Fig. 1, is subjected to direct tension, T, and a bending moment, M, which is produced by the eccentricity, $e = \frac{M}{T}$. Since concrete cannot resist tension, the only stresses in the section are those in the steel. For equilibrium, the algebraic sum of all forces acting on the section must equal zero. Therefore,

$$T = A_s f_s + A'_s f'_s \quad \text{..........(1)}$$

Taking moments about the center of the upper steel rods,

$$A_s f_s = T \frac{\left(d - \frac{d_t}{2} + e\right)}{2\,d - d_t} \quad \text{..........(2)}$$

In the same manner, taking moments about the center of the lower steel rods:

$$A'_s f'_s = T \frac{\left(d - \frac{d_t}{2} - e\right)}{2\,d - d_t} \quad \text{..........(3)}$$

[2] "Concrete, Plain and Reinforced", by F. W. Taylor, S. E. Thompson, and Edward Smulski, Vol. 1, Edition 4, N. Y., John Wiley & Sons, Inc., 1925, p. 137.

Equations (1), (2), and (3) are used to determine the stresses in the two groups of steel. However, since there are, altogether, four unknowns—A, f_s, A'_s, and f'_s—and only three equations, it is necessary to make an additional assumption before these equations can be solved. Many designers arbitrarily make the area of steel in the upper and lower groups the same; or (which would be more economical), the unit stress in the steel of the upper and lower groups may be assumed to be equal.

If $A_s = A'_s$, the required area of steel in one group will be governed by the maximum allowable tensile stress in steel; thus:

$$A_s = \frac{T}{f_s} \frac{d - \frac{d_t}{2} + e}{2\,d - d_t} \qquad \text{(4)}$$

in which f_s is the maximum allowable stress in the steel. In the other group the steel will be under-stressed; thus:

$$f'_s = \frac{T}{A_s} \frac{d - \frac{d_t}{2} - e}{2\,d - d_t} \qquad \text{(5)}$$

Comparing Equations (4) and (5), it is apparent that when the pull is axial and the eccentricity is zero, the stress in both groups of steel rods is equal. As the eccentricity increases from $e = 0$ to $e = \left(d - \frac{d_t}{2}\right)$, it becomes more and more wasteful to place the same quantity of steel in both groups. When e becomes equal to $d - \frac{d_t}{2}$, the line of direct tension coincides with one group of steel rods, the stress in the other group equals zero, and there is no need of placing the two groups. In such case, or when e nearly equals $d - \frac{d_t}{2}$, the use of two equal groups of steel will make the total reinforcement in a member almost twice as great as if the steel had been designed for $f_s = f'_s$.

Should the same unit stress be used in both groups of bars (that is, $f_s = f'_s$), A_s is the same as would be determined by Equation (4), and A'_s becomes:

$$A'_s = \frac{T}{f_s} \frac{d - \frac{d_t}{2} - e}{2\,d - d_t} = A_s \frac{d - \frac{d_t}{2} - e}{d - \frac{d_t}{2} + e} \qquad \text{(6)}$$

Again, when $e = d - \frac{d_t}{2}$, the line of direct pull coincides with one group of rods, and no steel is required in the other group. When e is greater than $d - \frac{d_t}{2}$, Equations (1) to (6) are not applicable because part of the concrete begins to act in compression, and, if two groups of steel are used, one group also is in compression.

Case 2.—Part of the Section Under Compression.—This condition is created when the line of direct pull falls outside the steel reinforcement in the member.

The steel may be placed either in the tension face alone, or in both the tension and the compression faces of the member. By placing it only on the tension side, a design is produced in which steel acts in tension and concrete in compression, the combination that usually results in greatest economy.

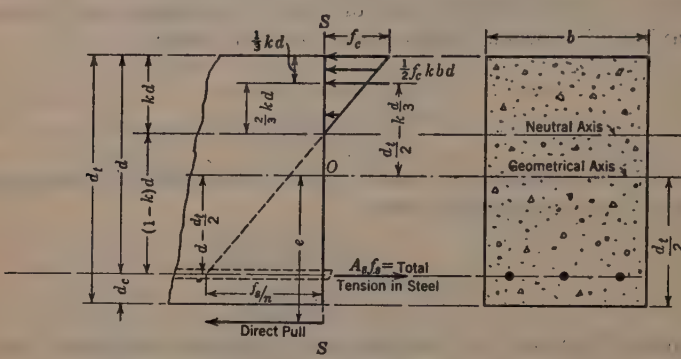

FIG. 2.

A thin section of concrete beam, *S*-*S*, in Fig. 2 (an independent structural element), is acted upon by the internal forces of compression in the concrete, the tension in the steel, and the external force of direct pull applied at a distance, *e*, from the geometrical center of the section.

Since Section *S*-*S* is in static equilibrium, the sum of the moments of all forces acting upon that section must be equal to zero. Taking moments about Point *O*:

$$A_s\, f_s \left(d - \frac{d_t}{2}\right) - T\, e + \frac{1}{2}\, f_c\, k\, b\, d \left(\frac{d_t}{2} - \frac{1}{3}\, k\, d\right) = 0 \quad \ldots\ldots(7)$$

The sum of all horizontal forces acting on the section must equal zero:

$$T - A_s\, f_s + \frac{1}{2}\, f_c\, k\, b\, d = 0 \quad \ldots\ldots(8)$$

Multiplying Equation (8) by *e* and equating to Equation (7):

$$A_s\, f_s \left(d - \frac{d_t}{2}\right) - T\, e + \frac{1}{2}\, f_c\, k\, b\, d \left(\frac{d_t}{2} - \frac{1}{3}\, k\, d\right)$$

$$= A_s\, f_s\, e - T\, e - \frac{1}{2}\, f_c\, k\, b\, d\, e \quad \ldots\ldots(9)$$

Substituting in Equation (9):

$$f_s = n\, f_c\, \frac{1 - k}{k} \quad \ldots\ldots(10)$$

and,

$$A_s = p\; b\; d \quad \ldots\ldots(11)$$

and solving for $\frac{e}{d}$:

$$\frac{e}{d} = \frac{2\,p\,n\,(1-k)\left(1-\frac{d_t}{2\,d}\right)+k^2\left(\frac{d_t}{2\,d}-\frac{k}{3}\right)}{2\,p\,n\,(1-k)-k^2} \qquad (12)$$

Substituting,

$$f_c = f_s \frac{k}{n\,(1-k)} \qquad (13)$$

and Equation (11), in Equation (8), and transposing:

$$\frac{T}{f_s\,b\,d} = p - \frac{k^2}{2\,n\,(1-k)} \qquad (14)$$

Equations (12) and (14), solved simultaneously, will give the values of k and p, the latter being the value sought. The remainder of the values in these two equations must be determined or assumed before the final solution.

Values of d_t and d—the total and the effective depths of the member—must be assumed before the stress distribution can be analyzed. This assumption is usually based on precedent or on the judgment of the designer. The value of b (Fig. 2) may be the full width of the member, in which event it is determined by trial, the same as d_t and d, or it may be taken as a unit width, usually 1 ft. Values of T and e are found from the analysis of stress distribution in the structure, thus, $e = \frac{M}{T}$.

The value of n which designates the ratio between the modulus of elasticity of concrete and the modulus of elasticity of steel, depends on the quality of concrete. Normally, it is assumed to vary between $n = 8$ and $n = 15$, which corresponds to $E_c = 3\,750\,000$ and $2\,000\,000$ lb per sq in., respectively. The value of f_s is determined by the grade of steel used and the purpose for which the structural member is designed, allowing an adequate factor of safety.

The two unknowns remaining are k and p. For any given condition of loading, dimensions, and stress distribution, the required amount of steel can be determined. However, the solution of Equations (12) and (14) is laborious, and, for this purpose, the writer has devised a set of diagrams, an example of which is reproduced on Fig. 3 for discussion. These diagrams were based on values of n equal to 15 and 10, respectively. In solving design problems by means of such curves the procedure is, as follows:

(1) Determine the bending moment, M, and the direct pull, T, for the member in question;

(2) Select trial values for d_t and d, unless they were tentatively chosen previously;

(3) Determine the value of $e = \frac{M}{T}$;

(4) Find the value of $\frac{e}{d}$;

(5) Assume a value of f_s, the unit stress in steel;

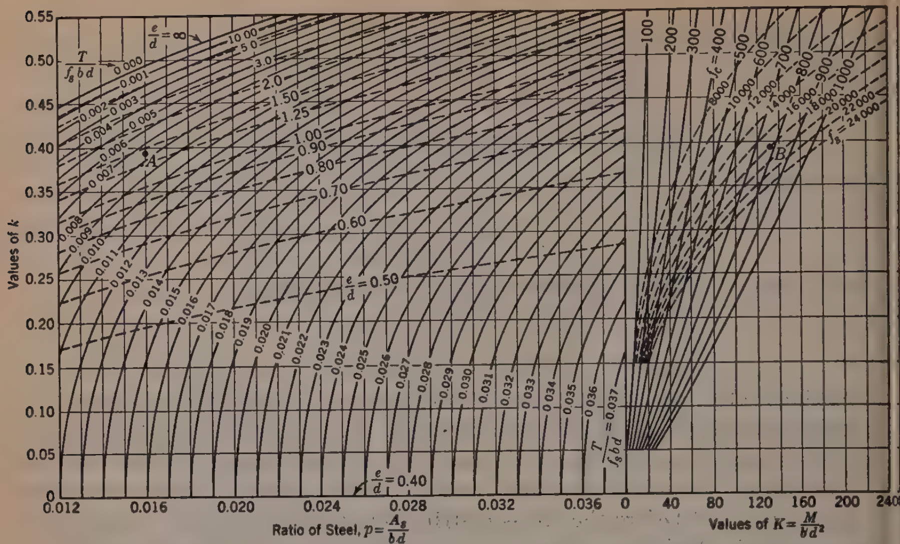

FIG. 3.—ECCENTRICITY, e, MEASURED FROM GEOMETRICAL CENTER OF CROSS-SECTION.

(6) Find the value of $\frac{T}{f_s b d}$;

(7) Find the ratio, $a = \frac{d_c}{d}$, of depth of cover on the steel to the effective depth of the beam;

(8) From a set of diagrams, such as Fig. 3, with the value of n in the problem, find the set of curves for a, closest to the value determined in Step (7);

(9) On Fig. 3 find the intersection of the two curves representing the values of $\frac{e}{d}$ and $\frac{T}{f_s b d}$ found in Steps (3) and (5);

(10) Directly below read the ratio of steel, p, to use for this particular condition; and

(11) Determine the area of steel, $A_s = p\ b\ d$, in section.

Incidentally, the value of k is determined by the same point of intersection as that found in Step (9). In order to find the unit compressive stress in concrete, follow a horizontal line from the point of intersection (found in Step (9)) to the right until its intersection with a curve representing the value of f_s assumed in Step (5). Finally, read or interpolate the value of f_c.

In most cases, the intersection of the curves for $\frac{e}{d}$ and $\frac{T}{f_s b d}$ will fall on the chart, which will give a definite solution of the problem. When the ratio, $\frac{e}{d}$, is very large the bending moment is very large in respect to direct tension. When this ratio approaches infinity, the case approaches simple bending and can be solved by means of simple bending formulas. However,

when both $\frac{e}{d}$ and $\frac{T}{f_s\, b\, d}$ are large, the value of k may be very high, and the unit compressive stress in concrete may be excessively high for any reasonable stress in steel. It may then be necessary, in most cases, to re-design the section.

When it approaches the lower values on the chart the ratio, $\frac{e}{d}$, becomes quite small, which indicates that the line of resultant direct pull approaches the line of steel. In this case, the direct tension, or the value, $\frac{T}{f_s\, b\, d}$, determines the amount of steel to use. Should the value of $\frac{e}{d}$ be smaller than the lowest value shown on the chart, the resultant direct pull falls within the effective section of the beam, and steel should be placed in both faces of the beam. In such a case, Equations (1) to (6) presented in Case 1 should be applied.

Should the intersection of the two curves fall outside the limits of the chart, it would mean that the case is of an unusual character as far as the percentage of steel or the relationship of stresses is concerned and that further analyses or re-designing may be necessary.

Returning, again, to Equations (12) and (14): When $k = 0$, there is no stress in the extreme compression fiber of the concrete. Substituting $k = 0$ in Equation (12):

$$\frac{e}{d} = 1 - \frac{d_t}{2\, d} \quad \text{.......................(15)}$$

or,

$$e = d - \frac{d_t}{2} \quad \text{.......................(16)}$$

In other words, this condition occurs when direct pull coincides with the steel reinforcement (see Fig. 2). Substituting $k = 0$ in Equation (14):

$$\frac{T}{f_s\, b\, d} = p \quad \text{.......................(17)}$$

or,

$$T = f_s\, p\, b\, d = f_s\, A_s \quad \text{.......................(18)}$$

that is, the quantity of steel is determined by the direct pull and the unit stress.

When the direct pull is zero, Equation (14) may be written thus:

$$k = \sqrt{2\, p\, n + (p\, n)^2} - p\, n \quad \text{.......................(19)}$$

which is a familiar relationship in the case of simple bending, when the eccentricity equals infinity, and the left side of Equation (12) is obviously equal to infinity; the right-hand expression of Equation (12) can be equal

to infinity only in case the denominator is equal to rezo, which again yields Equation (19). The use of the diagrams may best be illustrated by an example.

Example 1.—Determine the area of steel to use in a precast concrete pipe of 12.0-ft inside diameter. Under an internal pressure the top of the pipe is to be 10 ft beneath the surface and water will flow through it under a pressure head of 50 ft. The thickness, t, of the shell equals 12 in.; the steel reinforcement is covered with 2 in. of concrete, and the effective depth is 10 in. The pipe is embedded in concrete along the lower quadrant of its circumference. The severest condition of stress is at the bottom where $M = 19\,230$ ft-lb and $T = 16\,530$ lb. Assume $E_c = 2\,000\,000$ lb per sq in., which means that $n = 15$.

By following the procedure indicated previously: By Step (3) the eccentricity, $e = \frac{M}{T} = \frac{19\,230}{16\,530} = 1.163$ ft $= 14$ in.; and by Step (4), $\frac{e}{d} = \frac{14}{10} = 1.40$; in the case of combined bending and tension where concrete is under compression, select $f_s = 18\,000$ lb per sq in. (Step (5)). Then by Step (6), $\frac{T}{f_s\, b\, d} = \frac{16\,530}{18\,000 \times 12 \times 10} = 0.0076$; by Step (7), $a = \frac{d'}{d} = \frac{2}{10} = 0.20$. Steps (8), (9), (10), and (11) require the use of Fig. 3 for the case, $a = 0.200$, and $n = 15$. In this case (a member subjected to bending and direct tension, combined), the eccentricity, e, is measured from the geometrical center of the gross section. The intersection of two curves (Step 9), $\frac{e}{d} = 1.40$ and $\frac{T}{f_s\, b\, d} = 0.0076$, is at Point A, Fig. 3. Directly below the intersection (Step (10)) read the ratio of steel, $p = 0.016$. The area of transverse steel is $A_s = 0.016 \times 12 \times 10 = 1.92$ sq in. per lin ft (Step (11)).

The value of k for this condition is approximately 0.395. Following the horizontal line from Point A to the right until its intersection with the curve, $f_s = 18\,000$, $f_c = 795$ lb per sq in. at Point B, Fig. 3. Should it be desirable to use steel reinforcement in both the compression and the tension faces of the member, the compression steel will help the concrete under compression. Steel reinforcement could be used in the compression side to reduce the compression stress in the concrete or in order to provide for possible reversal of stresses.

Analysis and Solution Related to the Center of Steel Reinforcement in Tension Face

The foregoing example, depending for quick solution on sets of diagrams of the type of Fig. 3, refers to the geometrical axis of the beam as the center of moments. This leads to consideration of a depth of cover on the steel and necessitates a number of charts for various ratios of the depth of cover on steel to the effective depth of beam.

It may be considered that the centroid of the tension steel is the true center of stress in the beam subjected to tension and bending. This considera-

tion eliminates the necessity for several charts, but changes the value of eccentricity by an amount equal to $d - \frac{d_t}{2}$.

Fig. 4 illustrates a condition similar to that of Case 2, in which part of the section is under compression and steel is placed in the tension face only. All the notation and assumptions previously mentioned will apply in this case with one exception, namely, the eccentricity is measured from the center line of the steel in the tension face and is designated e'.

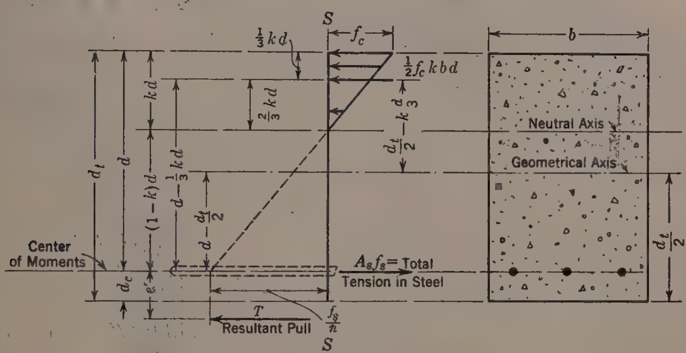

FIG. 4.

Taking moments about the centroid of the tension steel, for condition of equilibrium:

$$T\,e' = \frac{1}{2}\,f_c\,k\,b\,d \times \left(d - \frac{1}{3}\,k\,d\right) \qquad (20)$$

Since the sum of all horizontal forces must be equal to zero:

$$T - A_s\,f_s + \frac{1}{2}\,f_c\,k\,b\,d = 0 \qquad (21)$$

Solving for $\frac{e'}{d}$ after transpositions and substitutions:

$$\frac{e'}{d} = k^2\,\frac{\left(1 - \frac{k}{3}\right)}{2\,p\,n\,(1 - k) + k^2} \qquad (22)$$

Equation (21), with substitutions and transpositions similar to those of Case 2, could be rewritten:

$$\frac{T}{f_s\,b\,d} = p - \frac{k^2}{2\,n\,(1 - k)} \qquad (23)$$

A simultaneous solution of Equations (22) and (23) will give the values of k and p.

A complete set of curves such as that shown in Fig. 5 obviates the necessity of laborious solutions and, similarly, to diagrams such as Fig. 3, gives a quick and accurate method of determining the percentage of steel

reinforcement in a member subjected to combined tension and bending moment. The writer has constructed diagrams for values of n equal to 8, 10, 12, and 15. Only one set of curves is necessary for the solution of a problem, but one must be careful to use eccentricity relative to the center of reinforcement in tension. Following standard procedure, Steps (1), (2), (3), (5), (6), (10), and (11) are exactly as before.

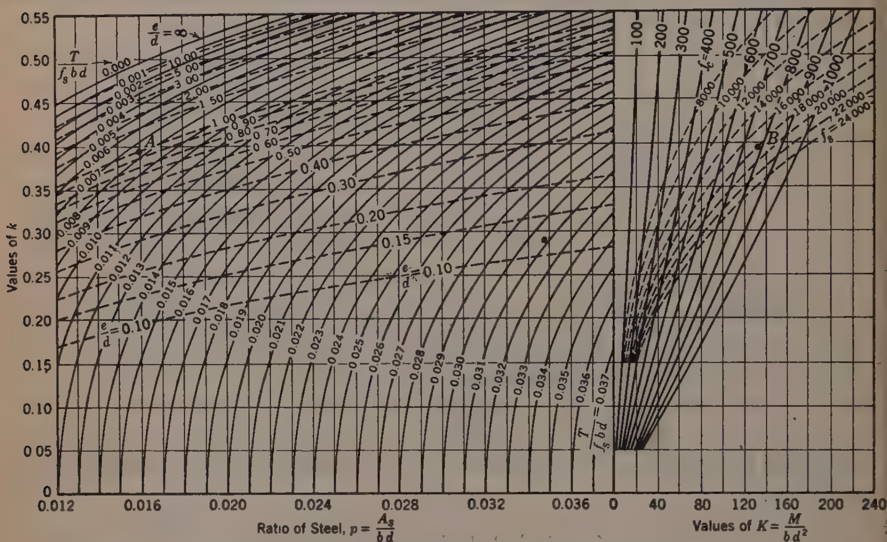

FIG. 5.—ECCENTRICITY, e, MEASURED FROM THE CENTER OF THE STEEL ON THE TENSION SIDE; $n = 15$.

In Step (4), determine $e' = e - d + \frac{d_t}{2}$; and from this find the ratio, $\frac{e'}{d}$. In this case Step (8) constitutes selecting a set of diagrams, such as Fig. 5, with the given or assumed value of n; and (see Step (9)), on this diagram, find the intersection of the two curves representing the values of $\frac{e'}{d}$ and $\frac{T}{f_s\, b\, d}$ found in Steps (4) and (6). The value of k and the unit compressive stress in the concrete are determined precisely as in Case 1.

Example 2.—Using the same problem as in Example 1, $M = 19\,230$ ft-lb; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; $d = 10$ in.; and $d_c = 2$ in. As before, the eccentricity relative to the geometrical center of the section (Step (3)) is $e = \frac{M}{T} = \frac{19\,230}{16\,530} = 1.163$ ft $= 14$ in. The eccentricity relative to the center of the steel reinforcement in tension (Step (4)) is $e' = e - d + \frac{d_t}{2}$ $= 14 - 10 + 6 = 10$ in.; and, consequently, $\frac{e'}{d} = \frac{10}{10} = 1$. Assume a unit tensile

stress in steel (Step (5)) of $f_s = 18\,000$ lb per sq in.; find the value, $\frac{T}{f_s\, b\, d}$ (Step (6)) $= \frac{16\,530}{18\,000 \times 12 \times 10} = 0.0076$; on Fig. 5 (Step (9)), find the intersection of two curves, $\frac{e'}{d} = 1.00$ and $\frac{T}{f_s\, b\, d} = 0.0076$ at Point A; directly below the intersection read the ratio of steel (Step (10)), $p = 0.016$; and (Step (11)), the area of steel required $= A_s = 0.016 \times 12 \times 10 = 1.92$ sq in. per lin ft of pipe.

As in Example 1, following the horizontal line from Point A to the right until its intersection with the curve, $f_s = 18\,000$, the unit compressive stress in the steel is $f_c = 795$ lb per sq in. at Point B, Fig. 5.

It is to be noted that the result of this solution is exactly the same as that obtained from diagrams such as Fig. 3 which are based on eccentricity relative to the geometrical axis of the center.

Diagrams of the type of Fig. 5, however, are superior to those of Fig. 3 in that they require only one set of curves for each value of n, giving a complete solution for most cases. However, in solving problems by these charts there is a tendency on the part of the student and the junior engineer to use the wrong value of eccentricity, and an emphasis on the importance of using the correct value is warranted.

The properties of curves, such as Fig. 5, are similar to those of the type of Fig. 3, but they deserve independent consideration. When the value of $\frac{e'}{d}$ is very large, it indicates that the bending moment is very large in respect to direct tension and dominates the solution. The larger the value of $\frac{e'}{d}$, the more indefinite is the intersection of the two curves determining the ratio of steel, and, in extreme cases, it may be necessary actually to solve Equations (22) and (23).

When the value of $\frac{e'}{d}$ approaches infinity, the condition approaches that of simple bending moment, and the problem is solved either by the formulas or by the diagrams. In order to solve a case of simple bending moment by means of diagrams such as Fig. 5, proceed as follows: Having determined the value of the bending moment, M, the width, b, and the effective depth, d, of the member in question, determine the auxiliary value, $K = \frac{M}{b\, d^2}$. Enter the right side of the diagram for the proper value of n with the value of K found in the previous step. Follow the vertical line to its point of intersection with the desirable or assumed stress lines for f_c and f_s, or with the limiting stress as the case may be. From this point of intersection follow the horizontal line to the left to its intersection with the curve, $\frac{T}{f_s\, b\, d} = 0$.

Vertically below the last intersection read the steel ratio, p. Determine the area of steel, $A_s = p\, b\, d$. No example is necessary to illustrate this case.

When $\frac{e'}{d}$ and $\frac{T}{f_s\, b\, d}$ are both relatively large, the value of K may be very high and the unit compressive stress in concrete may be excessively high for any reasonable stress in the steel. In most cases, it would be necessary to re-design the section by increasing the over-all dimensions, b and d, of the member, or by placing steel in the compression side in order to relieve the stress in the concrete, or both.

When $\frac{e'}{d}$ is small it means that the line of direct pull approaches the line of steel. The direct tension, or the value, $\frac{T}{f_s\, b\, d}$, determines the amount of steel to use. It may happen that the value of e' is negative, which is possible when the value of e, by Step (3), is smaller than $d - \frac{d_t}{2}$. This would indicate that the line of direct pull falls within the effective section and that the two layers of steel are needed. Equations (1) to (6) in Case 1 provide the solution.

Conclusion

A full appreciation of the advantages of the procedure outlined herein, can be obtained only by using, in practice (and over a reasonable period of time), the complete set of curves upon which the arguments of the paper are based. In offering a condensed outline description of the curves it is hoped that discussion will develop their value in principle and that suggestions for their improvement, for example, such as the desirable intervals of the n-curves, will be advanced. If it is conceded that complete sets of curves such as those described in this paper, would be valuable to structural designers in this field, should they be of the type demonstrated by Fig. 3, or Fig. 5? These and other questions could be answered in discussion.

APPENDIX

Notation

The symbols introduced in the paper are summarized herein, for convenience of reference, as follows:

$a = \frac{d_c}{d}$ = ratio of the depth of cover on the steel, to the effective depth of the beam;

b = breadth of beam;

c = distance from the neutral axis of a beam to the extreme fiber in compression; as a subscript, c denotes "concrete";

d = depth; effective depth of a beam, or the depth from the extreme fiber in compression, to the centroid of the steel reinforcement on the tension side of the beam; d_t = total depth of a beam; d_c = depth of cover on the steel reinforcement, measured from the centroid of the rods; $k\,d$ = distance from the extreme fiber in compression, to the neutral axis;

e = eccentricity of the resultant force, T, in respect to the geometrical axis of the entire section $\left(e = \frac{M}{T}\right)$;

f = unit stress; f_c = unit compressive stress in the extreme fiber of a concrete section; f_s = unit tensile stress in steel; f'_s = unit stress in the upper steel rods;

k = ratio of the depth of the neutral axis from the extreme fiber in compression to the effective depth of the beam; $k\,d$ = the distance from the extreme fiber in compression to the neutral axis;

n = the ratio of the moduli of elasticity, $\left(n = \frac{E_s}{E_c}\right)$;

p = ratio of steel area to the effective area of a beam, $\left(p = \frac{A_s}{b\,d}\right)$;

s = a subscript denoting "steel";

t = thickness; as a subscript, t denotes "total";

A = area; A_s = cross-sectional area of steel ($A_s = p\,b\,d$); A'_s = cross-sectional area of the steel in the upper part of a beam;

C = a subscript denoting "depth of cover";

E = modulus of elasticity; E_c for concrete and E_s for steel;

I = rectangular normal of inertia;

K = a constant, $\left(K = \frac{M}{b\,d^2}\right)$;

M = moment; bending moment at the section under consideration;

T = total tension; resultant pull due to external forces.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

REPORTS

CONSERVATION OF WATER

PROGRESS REPORT OF THE COMMITTEE OF THE IRRIGATION DIVISION[1]

The Committee on Water Conservation was appointed in 1929. Except for the annual statements of progress and program, the Committee has not made any report to the Irrigation Division since its organization. It now presents the results of its activities and its conclusions on several matters relating to water conservation.

The principal work of the Committee has consisted in the study of several matters relating to water conservation by sub-committees of its own members and sponsorship of two conferences on water conservation held in Los Angeles, Calif., in 1930 and 1935.

The membership of the Committee consists mainly of California engineers in order to reduce the expense of Committee meetings. In consequence, the Committee has directed its activities largely toward matters of water conservation of importance in California, and more particularly toward the problems of Southern California, which are acute and of direct public interest.

Sub-committees on the following subjects have been appointed: (1) Water Spreading; (2) Permeability of Soils in Relation to Water Conservation; (3) Bibliography Relative to Water Conservation and Allied Subjects; (4) Modifying the Physiographical Balance by Conservation Measures; (5) Absorption of Precipitation (Rainfall Penetration); and, (6) Legal Status of Ground-Waters and Return Water.

A paper entitled "Modifying the Physiographical Balance by Conservation Measures," by A. L. Sonderegger, M. Am. Soc. C. E., has been published by the Society[2] as the result of the work of the Sub-Committee.

Papers[3] covering the results of the Sub-Committee on Permeability of Soils have been prepared by Charles H. Lee, M. Am. Soc. C. E.; and of the Sub-Committee on Legal Status of Ground-Water, by Harold Conkling, M. Am. Soc. C. E. Similar papers are in preparation for the Sub-Committee on Water Spreading, by Kenneth Q. Volk, M. Am. Soc. C. E., and on the Precipitation Absorption, by Harry F. Blaney, Assoc. M. Am. Soc. C. E.

The Committee has also endeavored to assist in the programs relating to water conservation of the different research agencies active in Southern Cali-

NOTE.—Discussion on this report will be closed in March, 1936, *Proceedings*.

[1] Presented at the meeting of the Irrigation Division, Los Angeles, Calif., July 4, 1935.

[2] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 284.

[3] Publication pending in *Proceedings*, Am. Soc. C. E.

fornia by holding two public conferences in Los Angeles in March, 1930, and March, 1935. At these conferences papers on subjects relating to water conservation were presented, followed by field trips to areas in which experiments were being conducted. The purpose of the Committee in these conferences has been that of a clearing house for research programs and results in this field.

The 1930 Conference discussed mainly matters relating to the consumptive use of water by valley and mountain vegetation; the conference this year (1935) covered a broader field. On the recommendation of the 1930 Conference, a committee was appointed to prepare a program of correlated investigation by the various public agencies working in this field. Many of the lines of investigation recommended are now in progress, and, for some, the results are becoming available. It is hoped that similar results will follow from the 1935 Conference.

Papers presented at the 1930 Conference were mimeographed and distributed among those in attendance. Those for the 1935 Conference are now being edited and will be mimeographed as soon as funds are made available for that purpose. Many of the conclusions are included in the resolutions passed by the conferences. The data on which the Committee's conclusions rest are not included in this report due to limitations on its length; much of the supporting material is included in the papers presented at the conference.

Conditions affecting water conservation vary widely in different localities. Conditions in Southern California represent a combination of factors that is found in few other areas. There is probably no general area in the United States where the local water supply is as limited in quantity and as valuable for use. There are few localities where steep erodible drainage areas adjoin as highly developed valley lands. In few areas is the annual rainfall insufficient to maintain native vegetation larger than brush; and yet there are areas where the rainfall during parts of the year occurs in heavy storms resulting in excessive rates of run-off and erosion. This combination of factors results in an urgent need for the conservation of as much as possible of the local water supply for use as well as to prevent damage from its uncontrolled run-off. Although much of the Committee's work has been in relation to the problems of this area, it has attempted to distinguish its conclusions which have only a local application from those that have a more general use.

Evaporation

Evaporation from water surfaces has been extensively investigated and discussed. Much material on this subject assembled by the Special Committee on Irrigation Hydraulics of the Society has been published in *Transactions*, Volume 99 (1934). There was additional discussion of this subject at the 1935 Conference of the Water Conservation Committee.

Two lines of investigation have been followed in the study of evaporation. Engineers have made measurements of its amount and attempted to correlate the results with readily available climatic records, such as wind movement.

temperature, and humidity. Physicists have approached the subject on the basis of the solar energy received. Both lines of investigation are useful, and any complete study of the subject requires consideration of them.

Sufficient direct observations are now available to enable evaporation to be estimated closely enough for most water supply purposes. Except for reservoirs of shallow depth or long periods of carry-over storage, evaporation is not a major factor in water supply development. Present information on evaporation from water surfaces is now generally adequate for nearly all engineering requirements.

Observations on evaporation from water surfaces in pans or tanks are useful as a standard of comparison with evaporation from soils and transpiration from plants. Much experimental work in this field is under way, and the inclusion of observations of evaporation from water surfaces is an essential part of such programs. The variations in different months of the relationship between evaporation from pans to that from large water surfaces illustrate the need for such a reference base in such work.

To date, no dependable formula for evaporation applicable to large geographical areas from which data can be determined by means of existing weather records, has been developed. It is not probable that such a formula can be developed as evaporation depends upon other factors besides those which have usually been observed at Weather Bureau stations. Such a formula, if obtainable, would have the most ready application, as it could be used with present climatic records.

Evaporation cannot exceed the quantity of water that can be converted into vapor by the amount of solar radiation received; it is less than this limiting amount as losses in solar energy occur, and it is not all available to produce vaporization. Measurements of solar radiation have not been made extensively, and the application of results for one locality to other areas is difficult, due to this lack of observational record.

It is the Committee's conclusion that there is need for a thorough and extensive study of evaporation in which the two points of approach previously mentioned are combined. Many past experiments have been limited in scope or conducted for insufficient periods; there is little need, except for local use, for further observations of this type. A comprehensive study of all factors continued long enough to cover climatic variations is needed and would be well worth while. Such a study should include the points of view of both physics and engineering in order that both the theory and the applications may be represented. For such a study California offers favorable climatic conditions with its freedom from winter freezing and the large rates of loss during summer months.

In line with these conclusions, the 1935 Conference passed the following resolution with which this Committee concurs:

"*Whereas* there is need for a complete and thorough study of the factors affecting evaporation and their quantitative effects;

"*And Whereas* past experimental work has not been sufficiently extensive and thorough or has not been continued sufficiently long to secure adequate results;

"*Be It Resolved* that this conference consider past work in evaporation commendable in its intentions and results but inadequate for present needs;

"*Therefore, Be It Further Resolved* that the conference favors the undertaking of a complete and thorough research program on evaporation based upon adequate financial support by Federal, State and private agencies, so that it may be sufficiently extensive in scope and continued for sufficient time to meet the needs of the engineering and other scientific uses of its results."

Economic Use of Irrigation Water

Recent deficiencies in rainfall and run-off have resulted in increased interest in the determination of the minimum quantities of use for irrigation with which adequate yields can be maintained. Due to water shortages, some use has been less than the amounts formerly considered necessary. The effect of such shortages on the yield and future life of the orchards remains to be determined by future results.

The necessity of conserving the limited water supplies of Southern California in order that the maximum feasible area may be served is generally recognized. Much valuable investigational work in this field has been done and is still in progress by State and Federal agencies. Many of the results were presented and discussed at the Committee's two conferences. The Committee recommends the continuation of these investigations by the appropriate agencies. Prompt publication of results is also urged.

Among the results of the study of the best irrigation practice for different soils has been the demonstration of the advantage of using different frequencies of application and the need for flexible delivery schedules for service from the canals. Under past practice many canals have been operated under fixed delivery schedules with uniform periods between irrigations. Many canals in this area have little storage or surplus pump or canal capacity, and uniform deliveries are required to enable the area served to be supplied. The relative advantages to the users of flexible service on demand and the cost to the company of the changes necessary to enable such service to be furnished are individual problems for each canal system. The experience of the companies now modifying their delivery methods indicates that many canals will be able to supply at least a partial delivery on demand without excessive increases in the cost of operation. For some companies this may be accomplished by the development of auxiliary ground-water supplies at strategic points in the delivery system.

Title to Use of Underground Water

The principles governing the rights to the use of percolating ground-waters are being subjected to close scrutiny in several States as the result of the increased use of this source of supply during recent drought periods. A paper on this subject has been prepared by Mr. Conkling at the request of this Committee.[3] The subject was also discussed at the 1935 Conference.

Different systems of title to the utilization of ground-water are in use in different States. In most of the States principles in use in other jurisdictions were first adopted by the Courts. Later and more extensive use of groundwater in many of the States has shown that such principles are not adapted

to securing the best utilization of such percolating waters under the conditions of these States, and modifications of the earlier decisions have been made. Recent American decisions on ground-water have been directed largely toward establishing rules of law for such waters that could be applied to the physical conditions of use. This process is still in progress.

California has been a leader, both in the extent of use of ground-water and in the establishment of principles governing rights to its use. The principle of correlative rights, as established by the California Court in 1903, represents a real effort to find rules of law that would be equitable among different classes of use and still permit a full development of this natural resource. Experience with these principles under the many conditions of actual use has shown some shortcomings. Correlative rights as now recognized in California are an effective system where a ground-water basin contains a ground-water supply in excess of the needs of the overlying lands; the relative rights of owners of land which overlies the ground-water, and distant takers, permit a full use of the available supply.

The California principles of under ground-water law are not as well adapted to areas where the supply is less than the needs of the overlying lands. Unfortunately, this condition occurs in many areas. For these conditions each owner is entitled to his *pro rata* share of the available supply, and overlying owners first developing use have no protection against over-draft resulting from later use by other overlying owners. With the extensive projects now in contemplation for recharge of ground-water from distant sources, some basis of title to such artificial ground-water will be needed.

All principles regarding ground-waters in California are the result of Court decisions. Although eventually these precedents may yield to public necessity, there is usually a considerable time lag during which unfavorable conditions continue. The Committee considers that there is need in California for a reconsideration of this subject in the light of present knowledge and conditions.

Changes in present practices in the control of ground-water, if made at all, should be made only after careful consideration by all the interests concerned. The Committee is not, at this time, prepared to present final recommendations on this subject. It does, however, urge its consideration and offers its co-operation with others concerned in seeking to find the best means of accomplishing the desired result. It considers that a Constitutional Amendment, perhaps along the lines of the recent Amendment of the California Constitution regarding rights of riparian owners on surface streams, is probably necessary to effect such a change, and believes that efforts to draft and secure its adoption are justified by the present conditions.

Soil Erosion

The study of factors affecting soil erosion and methods for its control have come into much prominence during the past two years. Although many of the problems of soil erosion are related only indirectly to water conservation, the two subjects also overlap at many points. Recognizing this condition, this subject was included in the program of the 1935 Con-

ference. Papers in this field were presented by Harry E. Reddick, M. Am. Soc. C. E., Regional Director, U. S. Soil Erosion Service[4]; Lewis A. Jones, M. Am. Soc. C. E., Chief, Division of Drainage and Erosion Control, Bureau of Agricultural Engineering, U. S. Department of Agriculture, and Messrs. E. I. Kotok and Charles J. Kraebel of the Staff of the California Forest and Range Experiment Station.

The extent of the activity of Federal agencies in the field of soil-erosion control makes this subject an essential part of any present study of water conservation. From the large amount of experimental work now in progress many data are becoming available. Points at which soil erosion affects water conservation are touched upon in the other parts of this report.

Burning of Native Vegetation

Discussions of the effect of burning of the brush cover on the water supply obtainable from a drainage area have been frequent and prolonged in this and other parts of California. At times, the heat of discussion has approached that of such fires. Many differences of opinion still exist. The Committee desires to call attention to a few items affecting this question.

Vegetation on a water-shed does not conserve water as far as the total water yield is concerned, as the moisture consumed by transpiration is lost. However, vegetation may effect, materially, the character of the occurrence of the run-off, both in the amount of flow and in the amount of erosion which it causes. The extent of such effect on any drainage area is a local question dependent on the amount and occurrence of the rainfall, steepness of the drainage area, susceptibility to erosion, and other related factors.

In areas of small slope and infrequent heavy rains, burning may reduce transpiration and conserve water without causing floods or serious erosion. In the steep and easily eroded areas adjacent to Los Angeles, where heavy winter rainfall occurs at semi-frequent periods, experience has shown that a heavy storm following a burn results in highly destructive floods and erosion. Until other methods may be developed, experience shows that burning of the native vegetation on these local areas should be prevented as far as practicable and that such control of burning justifies drastic control of the use of these areas for other purposes. The Committee, however, desires to add a word of caution regarding the application of these conclusions to other areas, where the conditions governing run-off may be different or where the lands below the steeper drainage areas have adequate flood channels or lack valuable improvements that may be damaged. Conclusions based on conditions in the San Gabriel Mountains adjacent to Los Angeles may have only limited application in other areas having essentially different conditions.

Transportation of Débris

Associated with the problems of erosion prevention are those of the transportation of the resulting débris where erosion is not, or cannot be, prevented. The principles governing the transportation of silt and other débris by running water have not been fully determined. Plans for adequate control

[4] Formerly U. S. Soil Erosion Service, Dept. of the Interior, now Soil Conservation Service, U. S. Dept. of Agriculture.

of débris require an understanding of the basic principles governing débris movement and their application to the conditions of the eroded area.

Débris or mud flows have been measured in which there has not been sufficient moisture to suspend it completely. Better instruments and standardized methods of measurement are needed in order to secure the observations required for the determination of the laws of flow. The mud flows following heavy storms on steep eroding areas are much more viscous than those of streams carrying heavy suspended loads, and special methods are needed for their study.

Much work, both in laboratories and in the field is now under way on this subject. The Committee urges the continuation of such studies. Present activity in this field includes several agencies. It appears to the Committee that co-ordination of the Federal activities in this work may be needed, and the Committee recommends that the Federal Government should concentrate the responsibility for such work in an appropriate agency with provisions for its adequate support.

Water-Spreading and Flood Channels

Water-spreading for the purpose of increasing the percolation to the ground-water storage is a widely used and well established part of current practice in Southern California. By such water-spreading is meant the diversion of stream flow on to adjacent areas of pervious material where it may be held until absorption occurs. Such spreading enables a larger part of the stream flow to be absorbed in the ground-water basins which represent such an essential part of the storage systems on streams in this area.

There has been some tendency to regard water-spreading as a method of flood control. This is erroneous as the capacity of spreading works cannot be made sufficiently large to control major flood flows. At flood stages the water may have too high a silt content to be suited to spreading.

There has also been a tendency to encroach on natural flood channels by actual construction along their course or by failing to keep the channels clear of vegetation. Such tendencies have been more active on streams for which storage or spreading works have been provided. The Committee considers that plans for flood control should include adequate provision for retaining the natural flood channels through the valley areas. Even with flood control by means of storage on the main stream, flash floods from lower tributaries or valley lands, particularly urban areas, may require nearly as large channel capacities as the uncontrolled stream.

Check Dams

No structures used in flood or débris control have been subject to as much controversy as check dams. As the term "check dams" is generally used in California it refers to those structures not exceeding 5 to 10 ft in height built in the beds of stream channels to reduce the grade and retard the flow. More substantial and larger dams having a material amount of storage capacity for débris or water are not included within the meaning of this term.

Many extravagant claims for the benefits of these structures have been made. Equally strong criticisms have been advanced. Check dams were fully discussed at the 1935 Conference. The popular interest in this subject fully justifies their serious consideration by engineers.

As a result of the experience with this type of structure the following conclusions have been reached by the Committee. These conclusions are taken directly from the following resolution, passed by the 1935 Conference, on "Check Dams in Mountain Areas of Southern California:"

"*Whereas* there have been constructed 4 500 check dams in the mountains of Los Angeles County, of a nominal height of from five to fifteen feet, about half of which have failed,

"*And Whereas* the term check or débris dam is generally applied to structures too small to create regulating storage,

"*And Whereas* there prevails an erroneous conception on the part of the public as to the function of check dams for flood control and conservation,

"*And Whereas* the subject of check dams has been discussed at the Water Conservation Conference in Los Angeles on March 13–14, 1935, and the following conclusions arrived at:

"1.—Check dams are essentially overflow dams and as such must be so constructed as to permanently withstand the scouring character of major floods.

"2.—Permanent débris dams in the mountain watershed properly designed and constructed, though seldom economically justifiable, will reduce the production of débris in erodible stream beds and may assist in stabilizing and consolidating slopes.

"3.—Check, or débris, dams do not assist materially in the conservation of flood waters.

"4.—Check dam systems do not affect the regulation of capital floods and cannot be recommended as flood control measures. They give an impression of false security to the residents of the lower lands.

"5.—The building of check or débris dams does not obviate the necessity for storm water or flood channels for capital flood discharge below the mouths of canyons.

"*Now, Therefore, Be It Resolved* that these conclusions be adopted as representing the results of the deliberations of this Conference and that copies of this Resolution be submitted to the authorities who are interested in the Control of Flood Water, Conservation of Flood Water, and Erosion Control."

This resolution represents a definition of engineering principles relating to check dams more than a resolution favoring or disapproving action regarding their use.

Investigations in Water Conservation and Control

To some extent, investigations of various factors affecting water conservation have been carried on by local districts or private agencies. However, the main burden of such studies has been, and should continue to be, carried by State and Federal agencies as the results have more than local application.

Much of the 1930 Conference was devoted to presentation by investigational agencies of the work under way and its results to that date. Similar presentations were made at the 1935 Conference. The great number of new

activities under the various branches of the Federal Government was brought out in the 1935 Conference.

As previously mentioned, the 1930 Conference concluded that there was a need for definition of the fields covered by different agencies and co-ordination of their programs to prevent overlap. A suggested program was included in the proceedings of the 1930 Conference. The Committee feels that the present need for similar co-ordination is fully as great as in 1930.

Extensive investigations of run-off and erosion have been begun in Southern California by the Federal Forest Experiment Station. Large costs have been incurred in the establishment and equipment of this work. Although much of the preparatory work has been done by agencies whose main purpose has been the creation of employment, a fine experimental set-up has been created. The Committee calls attention to the obvious fact that the value of this program will depend not upon the expense of the installation necessary for its initiation, but on the continued support for the observations over a relatively long period which will be necessary in order to secure adequate results. The Committee urges that, since the costs of installation have now been incurred, provisions should be made for the sustained support of the experimental work for a sufficiently long time to furnish records covering both wet and dry periods.

Papers were presented at the 1935 Conference on the results of intensive studies of run-off from small areas. Such studies have been made by local agencies as well as by divisions of the Federal Government. This work is being continued. The Committee considers that such work represents a commendable effort to analyze the factors affecting run-off. It appears to the Committee that there is need for a closer co-operation among the agencies in this field in order to avoid duplication and to make the results by different agencies more nearly comparable.

The authors of the papers at the 1935 Conference, describing the work on such experimental run-off plots, recommended the appointment of a committee to standardize such tests and to seek a solution to the following questions: (1) Effect of interception by brush and timber on run-off; (2) effect on run-off coefficients of different soil moisture contents; (3) effect of size of test plot; (4) relation of soil types to run-off coefficients; and (5) relation of test plots to large areas.

The Conference passed a resolution urging adequate financial support for such work and the appointment of a joint technical committee to correlate the investigations being carried out by different local agencies.

Respectfully submitted,

Committee on Conservation of Water,

ARTHUR L. SONDEREGGER, *Chairman,*
HARRY F. BLANEY, *Secretary.*
HAROLD CONKLING,
S. T. HARDING,
D. A. LANE,
CHARLES H. LEE,
WALTER E. SPEAR,
KENNETH Q. VOLK.

July 4, 1935.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

RATIONAL DESIGN OF STEEL COLUMNS

Discussion

By L. T. Wyly, M. Am. Soc. C. E.

L. T. Wyly,[58] M. Am. Soc. C. E. (by letter).[58a]—The method of analyzing steel columns, proposed in this paper, is the most rational that has been advanced to date. The problem has been defined concisely by B. R. Leffler,[59] M. Am. Soc. C. E., as follows: "The problem of the column is one of bending moment. The very fact that a column fails with an average unit stress below the strength of the material is evidence that it is impossible to centralize the load at every section, * * *." The parabolic formulas now generally advocated and widely used omit entirely (except as covered by a constant, representing assumed average values) the important factors of eccentricity and the value, c. Any solution of a bending problem which neglects these terms can be neither logical nor complete. The writer feels, however, that before the Engineering Profession can adopt Mr. Young's procedure it must have a more general knowledge of secondary bending actions, and more definite information regarding actual eccentricities to be expected on columns, as determined by careful and comprehensive field and shop measurements. Furthermore, the rational design of steel columns must necessarily take into account a number of factors not represented in the formula but which directly affect the strength of the column. Among the latter are the following:

(1) The form of section including its strength under local buckling action;

(2) Adequateness of design of connection, pin-plates, bracing, and other items unfortunately too often considered as secondary details; and,

(3) The relative degree of excellence to be expected in fabrication shops and in field construction in matters such as milling, setting shoes and grillages, and maintaining the structure in proper position during construction.

In short, it is the writer's conviction that a column formula should be tied up directly with specific clauses in design and construction specifica-

Note.—The paper by D. H. Young, Jun. Am. Soc. C. E., was published in December, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1935, by Messrs. William R. Osgood, Alfred S. Niles, J. F. Baker, and K. L. DeBlois; May, 1935, by Marvin A. Gray, Esq.; and August, 1935, by Messrs. R. G. Sturm and Marshall Holt, F. E. Turneaure, N. J. Durant, E. C. Hartmann, and Edward Godfrey.

[58] Asst. Engr., Div. of Highways, Springfield, Ill.

[58a] Received by the Secretary October 24, 1935.

[59] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 541.

tions, and the formula used with a fairly accurate idea of how these specifications will be followed. The writer desires to discuss briefly some of the evidence supporting this view.

It should be recognized at the outset that the problem is concerned with short columns primarily, or, at most, with intermediate columns. The slenderness ratio, in the plane of the truss, of main columns of heavy bridges will usually be 40, or less, except in the inclined end posts where it may be 60 or 75. In a light highway bridge, particularly if rolled wide flange beams are used as columns, the ratio may be as high as 100. The $\frac{L}{r}$ ratio normal to the truss will usually be less. In short, the Euler values do not enter into the problem.

Computed Eccentricities.—If eccentricities in the plane of the truss are to be determined by secondary stress analysis, the following factors should be taken into account: (*a*) For a large bridge with subdivided panels and heavy chords the secondary hangers or struts will be relieved of considerable primary stress by the beam action of the chords, with consequent reduction of secondary moments on the chords themselves; and (*b*) with railway loadings the local effect of heavy drivers is likely to be severe on the secondary moments in the chords, and an accurate solution may be very tedious.

The foregoing facts have been established by J. I. Parcel and G. A. Maney,[60] Members, Am. Soc. C. E. There is the further question of whether, at the present time, it would be advisable to entrust the matter of a secondary analysis of a truss, and consequent selection of a column formula for individual members, to the average designer. The writer's view is that much more work will have to be done on this problem and certain standards established by the profession before the method will be satisfactory for adoption in a specification. The same applies to eccentricities arising in a plane normal to the truss due to floor-beam or other rigid-frame action.

Measured Eccentricities.—With regard to initial crookedness of columns, the Final Report of the Board of Engineers of the Delaware River Bridge, between Philadelphia, Pa., and Camden, N. J., shows measurements taken on twenty-two columns varying in length from 29 to 41 ft. Initial crookedness of $\frac{1}{4}$ in. and more were found, and were just as great in the short members as in the long. The writer has taken careful measurements on about sixteen columns fabricated for a large highway bridge by a modern shop and has found initial crookedness of $\frac{1}{4}$ to $\frac{5}{16}$ in. These columns were as long as 100 ft, and the sectional areas up to 75 sq in. and most of them had central longitudinal diaphragms included in the section. The short columns showed as much eccentricity as the longer ones. It is the writer's belief based on the foregoing information, and on discussions with shop foremen and inspectors, (1) that initial crookedness is not necessarily a function of length or $\frac{L}{r}$; (2) that it is likely to be as much as $\frac{1}{4}$ in. for a short column; and

[60] "Secondary Stresses in Kenova Bridge", *Bulletin No. 4*, Eng. Studies, Univ of Minnesota, Minneapolis, Minn.

(3) that it should not exceed $\frac{3}{8}$ in., for a member 100 ft long if shop inspection is adequate. Furthermore, the writer would point out that it seems reasonable to believe that a column with a central longitudinal diaphragm or web-plate normal to the main webs should have less initial crookedness than one in which the webs are laced together, the shop work being easier in this respect. The writer would propose a clause in construction specifications limiting allowable initial crookedness of a column to an amount equal to 0.125 in. plus 0.0002 L (in which L is length of column, in inches), and making a greater amount than this cause for rejection.

There are important initial end eccentricities to which columns may be subjected, which are not due primarily to rigid frame or secondary bending action but to what may be termed accidental misfits in shop or field work. Probably the most frequent are:

1.—Eccentricities due to improper column alignment due to such causes as the following: (*a*) Improper fit of lateral bracing may pull members out of line transversely; and (*b*) settlement of one or more falsework supports during riveting work.

2.—Eccentricities due to misfits at ends of columns which depend largely or wholly on direct bearing for transmission of load.

It should be noted that the methods of computing the ultimate strength of columns subjected to initial eccentricities of this type are not included by Mr. Young, since the eccentricity (that is, end moment divided by column load) is not constant, as may be assumed with eccentricities in a truss arising from secondary bending.

Any column may be subjected to eccentricities due to improper alignment, and unless careful check is maintained of the position of a structure during the construction stages such errors may easily escape detection. An example in the writer's experience is shown in Fig. 26 in which a bridge column, shop riveted to a length of 100 ft, was pulled out of line $\frac{3}{8}$ in. in the field by an improper fit of lateral bracing. This was corrected by reaming the lateral connections. Some idea of the shears that may occur in a column by this kind of accidental eccentricity is given by the curves shown in Fig. 27. It is the writer's belief that errors of this type occur more often than is generally realized.

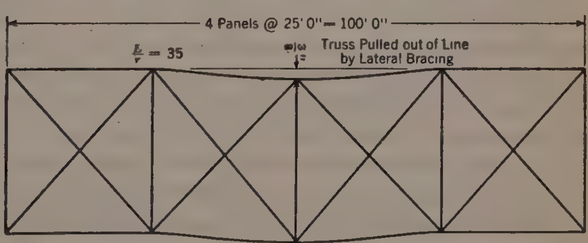

FIG. 26.

Columns subject to eccentric loads due to imperfect end bearings include some of the largest and most important compression members in engineering structures. Among them are: (1) All end posts or main vertical posts at piers that bear directly on a footing, on pins, on shoes, or on grillages; (2) main truss, or building columns that depend partly or entirely on milled ends for load transfer at splices; (3) trunnion columns supporting main trunnion bear-

ings or trunnion cross-girders in a bascule bridge, as well as columns supporting counterweights, or movable span and counterweights, in bascule or vertical lift bridges; and, (4) columns in movable bridges, that bear on main trunnions.

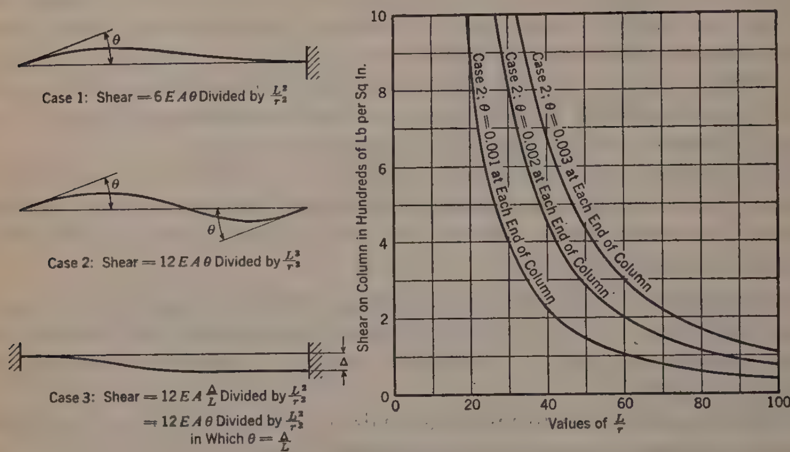

FIG. 27.

In through bridges most portal columns are in Class (1), and these may be subject to severe bending from lateral loads in addition to direct loads. A source of serious error in fit at tops of the columns of Class (3) may be the necessity for shimming between column top and base of bearing or of cross-girder. Frequently shims of varying thickness are used in this case. As it is manifestly impossible to get a thin shim which is perfectly flat, and as usually the weight of the bearings is not sufficient to iron out small waves in the shims, except in the case of the very largest bridges, it is evident that the bearing will not take its final position on the column until considerable dead load from the structure has been added, and at that time the actual error usually can not be detected or corrected. Where a cross-girder rests on the column the problem is further complicated by the fact that the girder will be subject to flexure—often much flexure—after the fit has been made and the dead load added to the girder. In the case of the columns of Fig. 28 special adjustments were made, after careful study, to take care of this matter. If there is any error in shop or field in making the center of the trunnion normal to the plane of rotation of the truss, the members of Class (4) and their connections are subject to racking or weaving.

In order to illustrate what may be expected from present practice in this matter there are shown, in Fig. 28 and Fig. 29, the results of careful measurements taken by the writer on the milled ends of trunnion columns on two different bascule bridges. In each case the grillage top plates were finished in the shop and were set in the field with great care. The measurements were taken after the columns had been lined in both directions and had been placed in proper position to receive the cross-girders or bearings supporting the movable leaf. Measurements of fit between grillages and columns were made by inserting thickness gauges between the top of the grillage plates and the base

of the column, under the main material of the column. Measurements on the tops of the columns in Fig. 29 were taken in a similar manner. All measurements were checked independently by other observers. Attention is

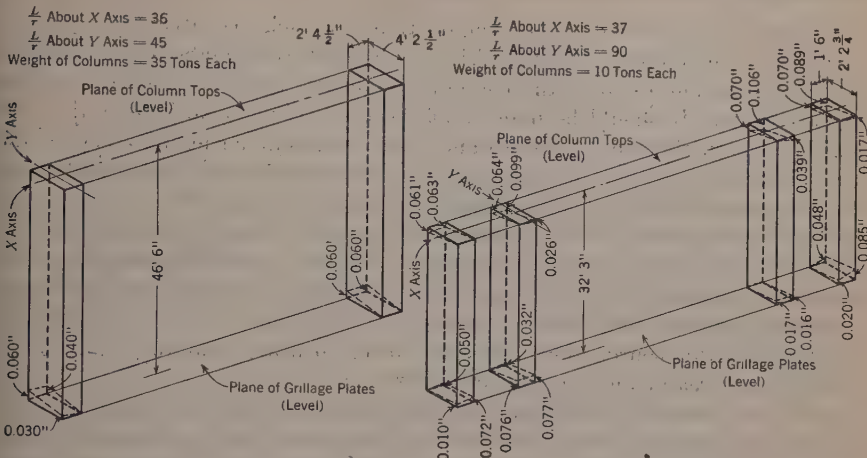

FIG. 28. FIG. 29.

called to the fact that in Fig. 28 the actual milled surfaces vary from the normal by an angle of 0.0012 radian, and that in Fig. 29 the surfaces vary from the normal by as much as 0.003 radian. In each case the weight of the columns themselves is sufficient to cause them to come to a firm bearing at the bearing edge or corner. It is apparent in the case of these columns that the error in milling bears no relation to the slenderness ratio of the column. The columns in these two bridges were fabricated by two different firms, each enjoying well established reputations for high grade work, and the work was inspected by professional shop inspectors. Furthermore, it should be noted that in the case of each bridge, the milling of the columns of the second leaf (not shown), was very satisfactory. In the case of the columns shown in Fig. 29, a proper bearing of the member on the grillage was obtained by tilting the columns. The Final Report of the Special Committee on Steel Column Research[61] shows even greater errors in end milling. Other instances of measured eccentricities at ends of columns due to errors in milling or in field work are, as follows:

(1) In the case of a large movable bridge carried on a heavy cross-girder which latter was seated through castings on heavy columns the writer found the load to be off center on one column by an amount of 2½ in., and off center on another by an amount of 1½ in. These measurements were checked by independent observers also. In the case of two other similar columns on the same structure the load was found to be located centrally.

(2) Column No. 5 of the Equitable Building, in Des Moines[62], Iowa, showed stresses corresponding to a rotation of the lower end of the column through an angle of more than 0.001 radian.

[61] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1412.

[62] *Bulletin* 72, Eng. Exp. Station, Iowa State Coll., Ames, Iowa, pp. 6 and 25, and Fig. 18.

(3) Column No. 4 of the American Insurance Union Building, at Columbus, Ohio[63], showed stresses corresponding to a rotation of the lower end of the column through an angle of 0.00138 radian.

Attention is directed to the implications of such eccentricities. In the case of all the columns shown on Figs. 28 and 29, the weight of the movable leaf was sufficient to force the column completely down to a bearing on the grillage plate. It should be noted that the errors due to milling at the base may well be duplicated at the top and that the eccentricities may act on the same side of the column, producing single curvature in the column and bending stresses at the middle (which may or may not be serious), or they may act on opposite sides of the column with resultant heavy shearing stresses. Fig. 27 illustrates this relation, showing the shears that will arise from this secondary bending at ends of members. The effect of direct column load, increasing bending and shears, is neglected. It is to be noted that the shears arising from this source alone are likely to be so heavy that they can not be taken by any reasonable amount of lacing.

It is the writer's belief, based on such information as the foregoing, that present methods of shop and field work and inspection may be expected to produce errors in end rotation of columns of 0.001 radian and that errors as large as 0.003 radian may occur easily. He would like to urge the necessity of a comprehensive investigation in shop and field of actual eccentricities occurring in structures.

The writer would also propose a special clause to cover milling for construction specifications and would suggest that definite limits to milling errors be set at an angle of 0.0005 radian and that larger errors than this be made cause for rejection. He would also call attention to the necessity for intelligent shop and field supervision in this work. Too often this work is entrusted to inspectors who have not adequate knowledge of the problem.

Local Failure.—The writer desires also to discuss briefly the question of local failure of the built-up short column and one or two factors producing such failure, together with its implications regarding the questions of column design raised in this paper. It is important to recognize that most of the models of large columns which have been tested and reported in available engineering literature have failed, not through integral action, but as a consequence of local distress. Consider, for example, the models of the second Quebec Bridge[64] having a stiffness ratio of $60 \frac{L}{r}$, or less; or, again, consider Models *U2M1* or *US2MS1* of the Memphis (Tenn.) Bridge.[65] For a number of years, prominent engineers even felt that it was impossible to design short columns that would not fail locally. One investigator has stated:[66]

"Now if one point be clearer than any other in regard to built up columns, it is that they do not act as solid or homogeneous columns. Unless improperly designed they always fail locally by the flange buckling between panel points,

[63] *Bulletin 40*, Eng. Exp. Station, Ohio State Univ., pp. 15, 16, 26, and Fig. 17.
[64] Final Report of Board of Engrs., Quebec Bridge, Vol. 1, pp. 197–215.
[65] *Technologic Paper No. 101*, National Bureau of Standards, U. S. Dept. of Commerce.
[66] "Columns", by E. H. Salmon, 1921, p. 185.

and it is their strength in this connection which determines their strength as a whole. If, of course, the column deflect in a direction perpendicular to the plane of the web bracing (lacing) only, it will act as a solid column, and no question of built up column arises."

However, the integral failure of the ten models of the Metropolis (Ill.) Bridge columns may well be said to have established conclusively the fact that it is entirely practicable to design short columns that will act as a unit, and it seems obvious to the writer that, until knowledge and practice of designing main section, end connections, transverse bracing, and other features too often considered as secondary details, is such that local failure is avoided, it is futile to rely upon any theory of integral action in column design. To illustrate, the failure of Model *U2M1* of the Memphis Bridge, through local failure of the forked ends, at a unit stress 25% below the yield point of the material, is probably a good indication of the strength of the forked ends, but not necessarily any criterion as to the strength of the member as a whole. The writer wishes to urge the proposition that the rational design of steel columns must necessarily include the design of any construction feature the premature failure of which can precipitate the collapse of the main member. There are a number of features of this class which are not adequately treated in design specifications at the present time. Prominent among them are: (1) Lateral bracing in the plane normal to the plane of the main webs of a column, this bracing commonly being composed of lacing at present; (2) the question of the proper relation between the size and stiffness of main flange angles and the web-plates; and, (3) the form of section as influenced by Factors (1) and (2).

With regard to the matter of lacing the writer feels strongly that a complete revision of present practice is urgent. He wishes to call attention to the following propositions:

(*a*) A column is subject to bending in the plane normal to the main webs, often quite as much as in the plane of the webs. Lacing does not form a proper element to resist this bending;

(*b*) Lacing on columns at present is usually much overloaded and the design loads specified are frequently about 50% of total stress carried; and

(*c*) As used at present, lacing induces serious deformations on the webs of the columns at splices, internal diaphragms, or other points of rigid support of the webs, with a direct material lowering of utilizable capacity of the main member.

The foregoing propositions will be considered in the order listed. In view of the relative flexibility of a system of lacing, as compared with a solid plate, it seems evident to the writer that the latter is essential in order to develop the maximum efficiency of a short column. To illustrate this point Fig. 30 shows the curves for deflection normal and parallel to main webs, of Model *U2U3* of the Municipal (St. Louis, Mo.) Bridge. It will be noted that at the useful limit point of the column (namely, about 50 000 lb per sq in.), the lateral deflection in the plane of lacing is about five times that in a direction parallel to the main webs. It is also to be noted that in the

case of Models *HC*1 and *MY*1 of the Metropolis Bridge[67] that the deflection at failure in the plane of the lacing, even with the solid plate in the section, is much larger than the deflection in the plane parallel to the main webs. Furthermore, most lacing is liable to abuse in handling, shipping, and erecting; it is inherently weak as a structural detail in that it is connected at the ends with one or two rivets. The difficulties arising from the failure of lacing are too well known to require much discussion.[68] The collapse at Panama, in 1915, during the official test, and under a smaller load than that for which it was designed, of the new 280-ton floating crane built for the Federal Government, was caused by such failure of lacing. With lacing replaced by a solid central plate normal to the main webs, the effect of a loose rivet or two would not be serious.

Present practice is to ignore completely the "participation" stresses induced in lacing, particularly double lacing, by deformation of the main column material under stress. Measurements on test models have shown that the lacing-bars adjacent to tie-plates, splice-plates, etc., are subjected to heavy induced stresses and, in fact, that these frequently exceed the stresses from transverse shears. In addition, the lacing on portal columns of through bridges is subjected to the shears arising from lateral loads on the structure which frequently are neglected in the design of the lacing. As a result, the lacing, at

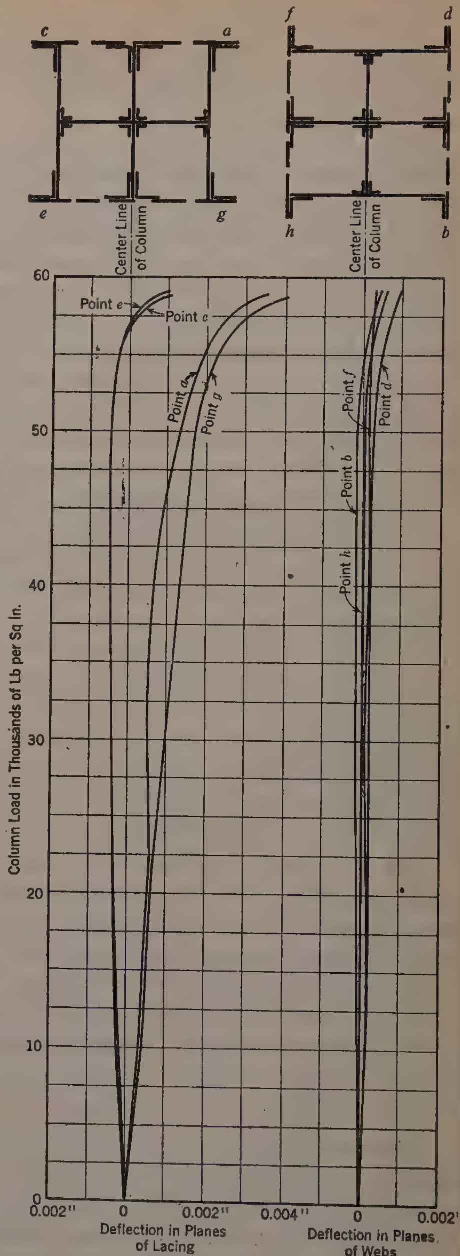

FIG. 30.

[67] *Technologic Paper No. 101*, National Bureau of Standards, U. S. Dept. of Commerce. Figs. 50 and 51.

[68] *Engineering News*, Vol. 73, May, 13, 1915, p. 918.

present, is often subjected to about twice the load for which it is designed. However, the main objection to present practice in lacing, from the writer's viewpoint, is that it induces distortions in the main column material at certain points and thus precipitates local failure of the member at a lower load than the material would otherwise carry. Lacing-bars acting as a pantograph, tend to force out of line the ribs of the column to which they are connected as the main material distorts axially under primary load. At points where the ribs are held rigidly in place, as at tie-plates, internal diaphragms, splice-plates, etc., this movement is prevented, but at the first connection adjacent to these points of support the lacing-bars are more or less free to move and, consequently, the distortion of the main material is greatest at these points. Careful measurement both of this movement and of the stresses induced in the webs of the column by it was made in connection with the tests on the models for the new Quebec Bridge.[69] The stresses in the main material at these points were found to be from 10 to 14% greater than elsewhere, and it is probable that the maximum stress was still greater since it was not possible to measure the stress at the sharp point of the local buckle. The number of test models of large built-up columns that fail locally as a result of this action is proportionately large. Conversely, the ten models of the Metropolis Bridge, were remarkably free from local failure, and all were braced with substantial lacing supplied with transverse ties at the ends of the lacing, and all included in the section central longitudinal web-plate normal to the main webs. An interesting piece of evidence in this matter is afforded by the results of tests on Models *TC*4, *TC*5, and *TC*6, of the 1912 Quebec tests,[70] in which the introduction of a central diaphragm plate in the section apparently increased the capacity of the members about 18 per cent. It is to be noted that in the case of Model *U*2*U*3, of the Municipal Bridge, local failure through lacing distortion was present; and yet the model developed a high percentage of efficiency. The presence of the central longitudinal diaphragm undoubtedly contributed very materially to this result.

In view of all the foregoing considerations, the writer would propose that solid plates or diaphragms centrally located, and normal to the main web-plates, be required on all main structural steel columns having a stiffness ratio of $60 \frac{L}{r}$, or less, and that preference be given the use of a solid plate instead of lacing wherever possible. Furthermore, he would propose that all lacing on main structural columns be supplied with transverse ties at all end connections to prevent lateral displacement of the main webs, and that all such lacing be proportioned to carry the additional stress induced by this arrangement. He would further favor a more definite requirement in the specifications that all lacing and transverse webs on columns subject to lateral stresses be proportioned for such additional loads. The induced stresses in the lacing are easily and quickly computed.[71] Distribution of shears

[69] Final Rept. of Board of Engrs., Quebec Bridge, Vol. 1, pp. 207–208.
[70] *Loc. cit.*, pp. 201–209.
[71] *Engineering News*, October 3, 1907, Vol. 58, p. 369.

between the lacing and the solid plate when both are used should rest on a rational basis of relative rigidity of the two systems considered as a truss or a beam over the length of the member.

The use of the aforementioned features is well established by precedent. Columns of this type were used throughout on the Metropolis Bridge and on the Kansas City Bridge over the Missouri River. This type of lacing was also reported to have been used in 1914 on the Hoang Ho Bridge, in China.[72] The writer has also used the foregoing features on several fairly large highway bridges. He was pleased to find that the shop foremen reported that the tie-bars were an advantage in assembling the material and that no particular difficulty was encountered with either the lacing or with the central diaphragm. For smaller structures rolled or built-up I-beam sections, with solid webs, will often prove very suitable.

In the writer's judgment, present specifications regarding the relative proportions of flange angles to web are entirely inadequate. The real importance of this matter should not be overlooked. The chords of the first Quebec Bridge[73] were built of material having an elastic limit of about 40 000 lb per sq in., and they were designed originally to take a working stress of 24 000 lb per sq in. and, later, computed to take a working stress of 27 000 lb per sq in. The bridge chords failed at an estimated stress of 18 000 lb per sq in.[74] The first models[75], failing through inadequate lacing, collapsed at a unit stress estimated to be about 22 000 lb per sq in. The second model, adequate in lacing, failed by the buckling of webs and flange angles between lacing connections at a unit stress of about 30 000 lb per sq in. This type of failure is also evidenced in the test made by A. N. Talbot, Past-President and Hon. M. Am. Soc. C. E., and Professor H. F. Moore[76], in 1910. By analogy to the case of web-plates supported on the edges and partly supported by stiffeners in the center it is apparent that the factors involved in this problem are the relative stiffness of the plate and the angles, the length along the flange angles between lacing supports, and the width thickness ratio of the web-plates themselves. Inasmuch as the Bryan theory[77], on which the buckling theory of column web-plates rests, assumes complete support against lateral displacement of the edges of the web, it is apparent that the angles must furnish this support in addition to being stable under their own load. The writer would like to urge the advisability of adequate tests along this line, the work to begin where the web-plate tests made for the Delaware River Bridge ended.

Finally, it seems to the writer that the foregoing considerations demonstrate conclusively that the form of section to be used must be taken into account when applying a column formula. The addition of a central longitudinal web-plate normal to the main webs will undoubtedly increase the unit

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[72] *Technologic Paper 101*, National Bureau of Standards, U. S. Dept. of Commerce, 1918, p. 13.

[73] *Engineering News*, Vol. 59, 1908, p. 422.

[74] *Loc. cit.*, pp. 404, 422.

[75] *Loc. cit.*, pp. 455–459.

[76] *Bulletin No. 44*, Univ. of Illinois Eng. Experiment Station, 1910, p. 34.

[77] "Problems Concerning Elastic Stability in Structures", by S. Timoshenko, *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 1010–1014.

stress that the member will carry, but will actually lower the allowable load if calculated on an $\frac{L}{r}$ basis alone, as is frequently the custom at present.

In conclusion, the writer desires to express his sincere appreciation and admiration for the remarkable work done by Mr. Young in developing a rational mathematical theory on what has been possibly the outstanding unsolved problem in structural engineering.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

LINE LOAD ACTION ON THIN CYLINDRICAL SHELLS

Discussion

BY MESSRS. U. FINSTERWALDER, AND F. W. SEIDENSTICKER

DR. ING. U. FINSTERWALDER[23] (by letter).[23a]—A clear and interesting contribution to the problems of cylindrical shells, and one that will be useful for the designing engineer, is contained in this paper. Various methods are in use to simplify cumbersome numerical computations in connection with the writer's theory.[8] Accurate stress values are obtained by the use of a simplifying hypothesis; namely, that for each case of unit loading the relationship between the work of deformation of the bending stress components and the direct stress components is constant for all proportions of a shell. With the help of this hypothesis it is possible to arrive at stress values for a given shell from stress values of cases that have been computed by the use of the rather cumbersome formulas of the writer's theory.

The writer neglected bending moments in the direction of the generatrix of the cylindrical shell. The solution as proposed by Mr. Schorer neglects, more or less, the bending resistance of the cylindrical shell in the direction of the generatrix, and also the influence of the stress components, S and T_2, on the work of deformation. For shells with widely spaced transverse stiffeners and of small widths the author's solution will be accurate. The rise of the cylindrical segment will then be comparatively small and the stress components, S and T_2, will not play an important part in the work of deformation. However, for very wide cylindrical shells, and although the distance between end stiffeners may be considerable, the influence of S and T_2 will be noticeable, and Mr. Schorer's solution may become inaccurate.

NOTE.—The paper by Herman Schorer, Assoc. M. Am. Soc. C. E., was published in March, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1935, by I. K. Silverman, Jun. Am. Soc. C. E.; and November, 1935, by Messrs. W. Flügge, and Anton Tedesko.

[23] Chf. Engr., Dyckerhoff & Widmann, A. G., Berlin, Germany.

[23a] Received by the Secretary November 11, 1935.

[8] "Die Theorie der zylindrischen Schalengewölbe System Zeiss-Dywidag", von Dr. Ing. Ulrich Finsterwalder, International Assoc. for Bridge and Structural Eng., Zurich, 1932; also, "Die querversteiften zylindrischen Schalengewölbe mit kreissegmentförmigem Querschnitt", von Ulrich Finsterwalder, *Ingenieur-Archiv*, Vol. 4, 1933, p. 43.

Professor F. Dischinger has worked on the problem, and has recently found the exact solution.[24] It is not more complicated than the writer's theory, although Professor Dischinger was able to eliminate the simplifying assumptions of the writer.

F. W. SEIDENSTICKER,[25] Esq. (by letter).[25a]—Knowing from experience that many readers are reluctant to follow through a series of difficult mathematical derivations, the writer fears that, in many cases, only the "Synopsis" and "Conclusions" of this excellent paper will be read. The wording of the concluding remarks may be misleading to the average engineer unless they are read as a part of the whole subject.

The fact that the proposed method of shell design is termed an approximation does not mean that it will be unsafe to use this method for practical applications, especially if one considers that the established formulas of structural design for use in standard steel and reinforced concrete structures involve a much higher degree of inaccuracy than the method presented by Mr. Schorer. His results are within a small percentage of the elastic theory. They become less accurate in cases where the stiffening supports are narrowly spaced. However, an exact solution in that case will be less necessary; it would be interesting to mathematical physicists, but less so to engineers, since structures of small spans usually do not justify a great amount of intricate engineering computations.

Good agreement between theory and practice was recently found in the results of full-sized loading tests[26] conducted during the demolition of cylindrical barrels at the Century of Progress grounds, in Chicago, Ill.

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[24] "Die strenge Theorie der Kreiszylinderschale in ihrer Anwendung auf die Zeiss-Dywidag-Schallen", von Prof. Dr. F. Dischinger, *Beton und Eisen*, Vol. 34, August, 1935. *et seq.*

[25] Cons. Engr., Chicago, Ill.

[25a] Received by the Secretary November 14, 1935.

[26] *Engineering News-Record*, November 7, 1935, p. 635.

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DISCUSSIONS

THE SHEAR-AREA METHOD

Discussion

By Messrs. Fang-Yin Tsai, and David A. Molitor

Fang-Yin Tsai,[24] Assoc. M. Am. Soc. C. E. (by letter).[24a]—In applying the principle of moment areas to determine the slope and deflection in beams under flexure, two different methods are available. One is the so-called slope-deviation method introduced by Professor Charles E. Greene[25], of the University of Michigan, and the other is the elastic-weight or conjugate-beam method presented by H. H. Westergaard[26], M. Am. Soc. C. E., and by Professors Otto Mohr[27], and H. Müller-Breslau.[28]

In the slope-deviation method, the following two well-known theorems apply when a beam with a varying moment of inertia is subjected to bending:

1.—The angle between the tangents drawn at any two points on the elastic curve is numerically equal to the area of the $\frac{M}{E\,I}$ - diagram between the ordinates at the two points; and,

2.—The deviation of any point from the tangent drawn at any other point on the elastic curve is numerically equal to the moment of the area of the $\frac{M}{E\,I}$ - diagram between the ordinates at the two points about the first point.

The foregoing two theorems have proved very useful for determining the slope and deflection in beams, particularly when the point of zero slope on the elastic curve is known or easily found as in the case of a cantilever beam

Note.—The paper by Horace B. Compton, Assoc. M. Am. Soc. C. E., and Clayton O. Dohrenwend, Jun. Am. Soc. C. E., was published in May, 1935, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: August, 1935, by Messrs. George E. Large, Samuel T. Carpenter, Roland H. Trathen, A. W. Fischer, J. Charles Rathbun, Harold R. Kepner, and Fred L. Plummer; and October, 1935, by Messrs. Albin H. Beyer, John M. Beatty, R. B. Peck, Ralph W. Stewart, C. W. Johnson and H. W. Birkeland, Garrett B. Drummond, and Harold E. Wessman.

[24] Prof. of Structural Eng., Dept. of Civ. Eng., National Tsing Hua Univ., Peiping, China.

[24a] Received by the Secretary November 2, 1935.

[25] *Michigan Technic*, June, 1910.

[26] "Deflection of Beams by the Conjugate Beam Method", *Journal*, Western Soc. of Engrs., Vol. 26 (1921), p. 369

[27] "Beitrag zur Theorie der Holz-und Eisen-Construktionen", *Zeitschrift* d. Arch. u. Ing. Vereines z. Hannover, Vol. 14 (1868), p. 19.

[28] "Beitrag zur Theorie des Fachwerks", *Zeitschrift* d. Arch. u. Ing. Vereines z. Hannover, Vol. 31 (1885), p. 418.

or a beam supported and loaded symmetrically. In the shear-area method, no such simple and elegant relationships exist between the shear-area and the slope and deflection as in the case of the moment-area method. Such relationships as these are prone to be so complicated and cumbersome as to have no practical value. The authors are to be commended for developing the shear-area method, not along the line of the slope-deviation theory, but along the line of the conjugate-beam theory (or "mathematical" beam, as named by the authors).

Although the foregoing discussion illustrates one of the advantages of the shear-area method as compared with the moment-area method, the use of the mathematical beam is not entirely advantageous. In the conjugate-beam method of moment areas, the slope and deflection at any section of a given beam corresponds to (and also equals), the shear and moment, respectively, at the same section of the conjugate beam loaded with the $\frac{M}{EI}$- diagram of the given beam, provided the supporting conditions of the conjugate beam have been properly chosen in accordance with those of the given beam. For instance, if the given beam has a fixed support at a certain end, at which both the slope and deflection equal zero, the same end of the conjugate beam must be supported so as to have both the shear and moment equal to zero; that is, the end must be free. Table 2 gives the proper supporting conditions for a conjugate beam corresponding to those of the given beam. (The notation

TABLE 2.—Supporting Conditions for the Conjugate Beam (The Moment-Area Method)

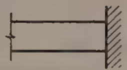
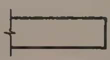
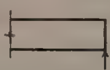
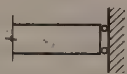
CASE	REAL BEAM		CONJUGATE BEAM	
1	Fixed	$\phi = 0$, $y = 0$	$V = 0$, $M = 0$	Free
2	Free	$\phi \neq 0$, $y \neq 0$	$V \neq 0$, $M \neq 0$	Fixed
3	Simple	$\phi \neq 0$, $y = 0$	$V \neq 0$, $M = 0$	Simple
4	Continuous	$\phi \neq 0$, $y = 0$	$V \neq 0$, $M = 0$	Hinge
5	Hinge	$\phi \neq 0$, $y \neq 0$	$V \neq 0$, $M \neq 0$	Continuous

is that of the paper.) With the supporting conditions for the conjugate beam properly chosen in accordance with Table 2, the shear and bending moment curves of the conjugate beam will be identical with the slope and deflection curves, respectively, of the given beam, not only in magnitude, but also in sign if a proper sign convention has been adopted.

The solution is very simple and straightforward, and the same procedure is equally applicable to beams with any supporting conditions and with the moment of inertia varying in any manner. No special devices such as the arbitrary introduction of two concentrated loads for the abrupt change in moment of inertia (Fig. 10) in the shear-area method are necessary.

In the shear-area method, the shear, bending moment, and slope at any section of the mathematical beam represent, respectively, the bending moment, slope, and deflection at the section of the real beam. As stated by the authors in Assumption (6), the supporting conditions of the mathematical beam must be determined in accordance with those of the real beam in the same manner as in the moment-area method. The writer has made a study of this phase of the problem the results of which are presented in Table 3.

TABLE 3.—Supporting Conditions for the Mathematical Beam (The Shear-Area Method)

CASE	REAL BEAM			MATHEMATICAL BEAM		
1	Fixed		$M \neq 0$ $\phi = 0$ $y = 0$	?	$V \neq 0$ $M = 0$ $\phi = 0$	?
2	Free		$M = 0$ $\phi \neq 0$ $y \neq 0$	?	$V = 0$ $M \neq 0$ $\phi \neq 0$	?
3	Simple		$M = 0$ $\phi \neq 0$ $y = 0$		$V = 0$ $M \neq 0$ $\phi = 0$	Two Rollers
4	Continuous		$M \neq 0$ $\phi \neq 0$ $y = 0$		$V \neq 0$ $M \neq 0$ $\phi = 0$	Fixed at Support
5	Hinge		$M = 0$ $\phi \neq 0$ $y \neq 0$		$V = 0$ $M \neq 0$ $\phi \neq 0$	Two Rollers

It is to be noted that the writer has failed to find the proper supporting condition for the mathematical beam (indicated by interrogation points in Table 3) to correspond with both the fixed and free ends of the real beam, since it seems impossible to devise an end-supporting condition with any physical meaning at which both the bending moment and slope will be equal to zero or not equal to zero at the same time. The authors have used a free end for the mathematical beam to correspond to the fixed end of the real beam, and *vice versa*, as shown in Figs. 8 and 9. This is incorrect. According to Assumption (5) of the paper, "the slope at any section of the mathematical beam represents the deflection at the same section of the real beam." At the free end of the mathematical beam the slope is not equal to zero; therefore, at the fixed end of the real beam, the deflection likwise must not be equal to zero, which is just opposite to the actual condition of a fixed end.

Similar inconsistencies may be cited against the authors' use of the fixed end for the mathematical beam to correspond to both the free and the simply

supported end in the real beam. When the real beam has a simply supported end, at which $M = 0$, $\phi \neq 0$, and $y = 0$, the same end of the mathematical beam must be supported by two rollers in a vertical plane, as shown in Table 3, so that the corresponding values are $V = 0$, $M \neq 0$, and $\phi = 0$. Although, any value can be assigned arbitrarily, to the shear, moment, and slope at any support of the mathematical beam, such a procedure will not only make meaningless the various types of conventional supports for the mathematical beam and the real beam, but will also cause serious confusion in the application of the method.

The conjugate-beam method of moment areas is based upon the similarity between the following two sets of well-known equations:

$$w = \frac{dV}{dx}; \text{ and, } \frac{M}{EI} = \frac{d\phi}{dx}$$

and,

$$w = \frac{d^2 M}{dx^2}; \text{ and, } \frac{M}{EI} = \frac{d^2 y}{dx^2}$$

Hence, if $w = \dfrac{M}{EI}$, then $V = \phi$ and $M = y$, the loading of the conjugate beam for finding the slope and deflection being the same. Similarly, the mathematical beam method of shear areas is based upon the similarity between the following three sets of well-known equations:

$$w = \frac{dV}{dx}; \text{ and, } V = \frac{dM}{dx}$$

$$w = \frac{d^2 M}{dx^2}; \text{ and, } \frac{V}{EI} = \frac{d^2 \phi}{dx^2}$$

and,

$$w = EI\,\frac{d^3 \phi}{dx^3}; \text{ and, } V = EI\,\frac{d^3 y}{dx^3}$$

Hence, the loading of the mathematical beam for finding the bending moment and deflection is the V-diagram only, and that for finding the slope is the $\dfrac{V}{EI}$-diagram, the loading of the mathematical beam for the three cases being thus not the same. In this connection, it may be also noted that Assumption (2) is not well founded, and that, if the V-diagram only is used as loading, the shear and slope at any section of the mathematical beam will not only "represent", but will also be numerically equal, respectively, to the bending moment and the deflection at the same section of the real beam.

Thus, despite the slight advantage in that the shear diagram is usually a somewhat simpler figure than the moment diagram for the same loading, the shear-area method has many inherent disadvantages as compared with the moment-area method.

DAVID A. MOLITOR,[29] M. AM. SOC. C. E. (by letter).[29a]—In their attempt to popularize the shear-area method of analyzing beams, the authors have not presented a very convincing claim for simplicity. Although it is true that shear diagrams are more easily drawn than moment diagrams, it does not follow that the former afford any material advantages over the latter in solving structural problems. Slopes and deflections are more directly derivable from moments than from shears, for which reason the shear-area method has not received much consideration.

In the stress analysis of simple beams or frames involving redundancy, the stresses are always evaluated from the moments at the critical sections to be analyzed. Therefore, the simplest and most direct method of approach is to deal with moments first, last, and all the time. The derivation of shears from known moments is relatively easy, whereas the reverse operation is attended with more difficulty. The slope of the elastic curve can be deduced from the moment curve by the method of area moments, without resorting to shear areas, by remembering that the slope at any point of a moment curve is represented by the moment derivative for that point. That is expressed in mathematical terms by the formula,

$$\frac{dM}{dx} = \tan \beta \qquad (190)$$

in which M is the moment at any point, m, acting over a small increment of beam length, dx, and β is the slope angle of the moment curve; but $\frac{dM}{dx} = V$ is the shear at the point, m, where the moment is taken.

By area moments, the deflection, y_m, at the point, m, is equal to the moment of the elastic weights, $M\,dx$, about the point, m, divided by $E\,I$. Therefore, the shear of the elastic weights on one side of the point, m, divided by EI, must equal the tangent of the slope angle to the elastic curve at the point, m. It should be noted that the elastic weights for negative moment areas are likewise negative.

By the same reasoning, the tangent of the slope angle of the elastic curve at any end support, A, must equal the end shear of the moment area, or the end reaction of the elastic weights divided by EI. Hence, calling V_m the shear of the elastic weights on one side of the point, m, of any beam, and R_a, the end reaction of the elastic weights at the support, A, the tangent of the slope angle, ϕ_m, of the elastic curve at the point, m, becomes:

$$\tan \phi_m = \frac{V_m}{EI} \qquad (191)$$

and the end slope at the support, A, becomes:

$$\tan \phi_a = \frac{R_a}{EI} \qquad (192)$$

[29] Structural Engr., Procurement Div., Public Works Branch, U. S. Treasury Dept., Washington, D. C.

[29a] Received by the Secretary November 6, 1935.

Equations (191) and (192) are applicable to either the graphic or the analytic methods of area moments. When the moment of inertia is variable, the elastic weights should be chosen as $\frac{M\,dx}{I}$ instead of $M\,dx$.

It is regrettable that the authors introduced the term "mathematical beam" without giving a precise definition, and also that they referred to what they term the conjugate beam method instead of simply the method of elastic weights. It seems inconsistent to attach qualifying terms to the beam itself, merely because it may be loaded either with actual loads, with moment areas, or with shear areas treated as elastic weights. More properly, the distinction should relate to the method of loading and not to the nature of the beams, which remains unchanged.

The authors have wisely limited their application of the shear-area method to beams. In dealing with rigid frames, the method could scarcely find favor when compared with the universal and elegant solution afforded by the use of Mohr's work equation.

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DISCUSSIONS

SOME LOW-TEMPERATURE CHARACTERISTICS OF BITUMINOUS PAVING COMPOSITIONS

Discussion

BY MESSRS. W. W. CROSBY, AND JOSEPH ZAPATA

W. W. CROSBY,[12] M. AM. SOC. C. E. (by letter).[12a]—In the years since the writer first proposed a scientific research in the field of bituminous paving compositions[13], these analyses have been greatly developed and improved in many respects. Mr. Skidmore is to be complimented on having pursued a line of great importance, but one that has been too much neglected in the past.

In the beginning of the introduction of bituminous highways it was known that a higher percentage of "free carbon" was desirable in tars for roofing purposes or sidewalks in New Orleans, La., than would be the case in Bangor, Me., because of the lower temperatures in the latter place and their "brittling" or cracking effects. Furthermore, the higher temperatures of New Orleans seemed to render a higher percentage of inert material desirable in order to prevent "running" at those temperatures.

Similarly, when asphaltic oils were offered for road use, too much "middle oil" was recognized as undesirable because of the instability of these materials —which lack "free carbon"—under high temperatures. As the latter phase was of much wider observance for a long time, the researches have been mainly along the lines of perfecting bituminous materials for the warmer climates. Now that the roads of the colder climates are being regularly cleared of snow, and their surfaces exposed to traffic, the importance of researches along the lines suggested by Mr. Skidmore should be appreciated.

The writer has previously expressed[14] his opinion on the value of ductility at 77° F. He agrees in the main with the summary of this paper, although he feels that some of the "facts" stated therein may be expressed somewhat loosely and thereby possibly may permit of dangerous interpretations; for instance, under Items (1) and (4) material might be used that, from its characteristics or quantities, would result in "bleeding" or "shoving" under traffic, in the hot suns of many localities.

NOTE.—The paper by Hugh W. Skidmore, Assoc. M. Am. Soc. C. E., was published in August, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by Messrs. Philip W. Henry, J. T. L. McNew, and Roy M. Green.

[12] Cons. Engr., Coronado, Calif.

[12a] Received by the Secretary November 8, 1935.

[13] "Sampittic Surfacing", by W. W. Crosby, *Transactions*, Am. Soc. C. E., Vol. LXIV (1909), p. 352.

[14] *Proceedings*, Am. Soc. for Testing Materials, Vol. XI (1911), p. 685.

JOSEPH ZAPATA,[15] ESQ. (by letter).[15a]—The author states that although the data reported in his paper pertain to sheet asphalt mixtures of the hot-mix type, sufficient work has been done with other compositions to demonstrate that typical characteristics disclosed by his studies are common to bituminous paving compositions in general. This conclusion does not agree with the results obtained by the writer in his experience and laboratory work.

In the first place, Mr. Skidmore gives no data on the stability characteristics of the aggregate mixture (sand-filler aggregate). Doubtless, this is due to the type of test that was used to evaluate the mixes discussed. During the last few years the writer has been working on the development of a stability test of wider range of application than either of the tests mentioned by Mr. Skidmore.[16] Under the conditions governing the test it was found that the addition of bitumen definitely lowers the resistance to displacement shown by the bare aggregate.

One of the conclusions reached by Mr. Skidmore is that the inherent characteristics and quantity of bitumen are much more important at low temperatures than at normal and higher temperatures in the pavement. The only characteristics given for the bitumens are penetration, ductility, cementation value, specific gravity, and the percentage soluble in carbon disulphide. The penetration has been limited to a range of 50 to 60; the available data are not sufficient to provide a clear understanding of the meaning of cementation value, specilc gravity, and the percentage soluble in carbon disulphide. the specific gravity and solubility values are not sufficient in themselves to establish inherent characteristics; therefore, the only characteristic available for making comparisons is the ductility. On the basis of the data shown in Table 1 the ductility seems to depend upon the origin of the petroleum. In view of the shear strengths shown on Fig. 2 is it possible, then, to assume that the choice of asphalts should be made on the basis of source?

Furthermore, Mr. Skidmore states that the ductility of the binders at the lower temperatures appears to be an important characteristic. His data do not confirm his statement inasmuch as the binder giving the highest shear strength is one that has no ductility at 41° F, 5 cm, 60 sec; also, in comparing the various binders it is noted that some show a difference of about 61% in ductility (41° F, 5 cm, 60 sec), with a difference of only 16% in shear strength. In making these comparisons, only data for temperatures of 41° F, 5 cm, 60 sec were considered for the following reasons: (1) A temperature of 41° F is practically that of the mean annual temperature for Wisconsin; (2) other ductility data available to the writer were obtained at a temperature of 39.2° F, 5 cm, 60 sec; (3) no data are given for ductilities at temperatures lower than 41° F, with a speed of 5 cm per min (although ductilities at 32° F are given, the conditions of test were changed, and thus an entirely new type of data are presented. What happens to the ductility, regardless of the method of test, when the temperature drops below 32° F?); and (4) in con-

[15] Asst. Materials Engr., State Highway Comm. of Wisconsin, Madison, Wis.

[15a] Received by the Secretary November 11, 1935.

[16] Preliminary Rept. has been pub. in *Proceedings*, Assoc. of Asphalt Paving Technologists, 1932, p. 105.

sidering low temperatures, weather records were kept in mind; for example, in Wisconsin one finds the following temperature conditions: During approximately 5.5% of the year the temperature is 0° F, or below; for about 39% of the year it is 32° F, or below; and during the remainder of the year it is above 32° F.

No proof has been furnished that the most destructive changes in pavements occur when the temperature is below 0° F, or between 0° F and 32° F. In an investigation conducted by the writer to determine the behavior of asphalts used as joint fillers, it was found that more damage was shown by the material during periods of alternate freezing and thawing than during periods of nearly zero weather.

Mr. Skidmore concludes that the ductility of the binder at the lower temperatures appears to be an important characteristic, and he recommends that the test for ductility be made at a standard rate of 5 cm per min and at a temperature of 4° C or 5° C. Making the safe assumption that by binder is meant a tar as well as an asphalt, Mr. Skidmore's data do not confirm his statement, especially if evaluations are to be made at 4° C or 5° C. The three binders showing high shear strengths at 4° C or 5° C have no ductility at those temperatures.

The design of the mixes was made on the basis of the voids theory; Mr. Skidmore does not indicate clearly, however, what steps were taken to balance differences in specific gravities. To illustrate the point, Table 4 offers a comparison. The question is raised in view of the statement that the common mixture was composed approximately of 10% bitumen, 18% filler, and 72% sand.

TABLE 4.—COMPARISON OF THE RATIONAL PERCENTAGE OF BITUMEN IN TWO BITUMINOUS MIXES

Material	Percentage by weight	Specific gravity	Rational proportion	Rational percentage
(a) MIX CONTAINING SPECIMEN 10				
Aggregate	90	2.662*	33.80	77.25
Bitumen	10	1.005	9.95	22.75
Total	.....	.....	43.75	100.00
(b) MIX CONTAINING SPECIMEN 5				
Aggregate	90	2.662	33.80	79.01
Bitumen	10	1.113	8.98	20.99
Total	.....	.....	42.78	100.00

* Computed from the specific gravity of the sand and the specific gravity of the mineral filler.

Mr. Skidmore fails to give data on the most important factor that should be considered in designing bituminous mixes, namely, the durability of asphalt cements. Without these data it is not possible to arrive at a complete understanding, or even to anticipate to a reasonable degree the possible behavior of mixes containing asphalts of various characteristics.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

FAILURE THEORIES OF MATERIALS SUBJECTED TO COMBINED STRESSES

Discussion

By Messrs. A. A. Eremin, and A. Floris

A. A. Eremin,[23] Assoc. M. Am. Soc. C. E. (by letter)[23a]—Tests of materials are not sufficient on which to base conclusions as to the validity of the theories of failure treated in this paper. Under "Conclusions", Professor Marin states that, "the precisely correct theory of failure is probably a combination of a number of these theories, depending upon the ratio, $\frac{S_1}{S_2}$".

Each theory with its basic assumptions and limitations has its individual historical and practical value. The theory of the strength of materials has always been a favorite subject of scientific research among mathematicians. In the first part of the Seventeenth Century the theories of elasticity were based on empirical values of either twenty-one or fifteen unknown constants. Discussion of some of these theories continues to the present.[24] Theorists and mathematicians have contributed numerous equations for computing ultimate forces sustained by various elastic members. It is an imposing task to apply some of these equations and theories in practice. The Euler formula for a column sustaining direct force was considered erroneous for a number of years. Only recently have the practical limitations of Euler's equations been understood. His equations have found wide application in the theory of the strength of materials.[25]

An interesting practical application of the theory of maximum shear stress failure has been made by Professor J. Fritsche.[26] He has developed the equations for computing the relation between direct stresses in a concrete test prism and ultimate stresses in concrete under combined stresses. Further-

Note.—The paper by Joseph Marin, Jun. Am. Soc. C. E., was published in August, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1935, by Messrs. J. J. Slade, Jr., T. McLean Jasper, and I. K. Silverman; and November, 1935, by Messrs. W. P. Roop, and H. F. Moore.

[23] Assoc. Bridge Designing Engr., California Highway Div., Sacramento, Calif.

[23a] Received by the Secretary October 29, 1935.

[24] "A Treatise on the Mathematical Theory of Elasticity", by A. E. H. Love, 1927, p. 13.

[25] "Strength of Materials", by S. Timoshenko, 1916.

[26] *Beton und Eisen*, April 5, 1935.

more, in the equations for computing the limiting stresses in concrete, Professor Fritsche introduced an exponential expression for variation of strain and stresses in concrete. However, as stated[27] correctly by the late George F. Swain, Past-President and Hon. M. Am. Soc. C. E., no one has yet proposed a reliable instrument for measuring the variation in stresses in material sustaining combined forces in space in such a manner as to verify the theories of failure.

A. Floris,[28] Esq. (by letter).[28a]—The critical examination of the existing theories of failure advanced by various investigators, and their correlation by means of a common set of co-ordinate axes, is the subject of this interesting paper. To the theories of failure treated, the writer wishes to add one more, proposed by Professor G. D. Sandel.[29]

The change in shape of a body under the influence of external forces, caused by the change in position of its particles relative to each other, can be considered as the measure of ultimate strength of the material. Consequently, the deformation of a body is determined by the slip of its particles along principal shearing planes and also by its volume change. The theory proposed by Sandel is based upon this reasoning. It is identical with the maximum shear theory when the influence of the intermediate principal normal stress is equal to zero. In general, however, this influence can be explained only if it is assumed that the volume change contributes also to the rupture of the material. At the limiting state of stress, therefore, the greatest sliding reaches a limiting value which afterward decreases linearly with the positive volume change (increase) of the body. Evidently, Sandel's theory of failure is a generalized maximum shear theory.

In the case of a bi-axial state of stress and with the author's notation, Sandel's theory of failure is expressed by,

$$(n + 1)\, s_1 + (n - 1)\, s_2 = 2\, s_8 \qquad (57)$$

in which, s_8 is the shearing stress at failure; and $n = \frac{s_c - s_t}{s_c + s_t}$ denotes the degree of brittleness of the material. For structural steel, $n = 0.04$; for cast iron, $n = 0.60$; and for concrete, $n = 0.88$.

For $n = 0$, Equation (57) expresses the maximum shear theory and for $n = 1$, the maximum stress theory of failure.

The bi-axial state of stress is easily extended to that of tri-axial stress by adding the term, $n\, s_3$ of the intermediate principal normal stress to the left-hand side of Equation (57).

The author is to be commended for bringing this important subject to the attention of engineers who, in practice, are accustomed to deal with allowable stresses.

[27] "Strength of Materials", 1924, p. 539.

[28] Dipl.-Ing., Los Angles, Calif.

[28a] Received by the Secretary November 18, 1935.

[29] "Ueber die Festigkeitsbedingungen: Ein Beitrag zur Loesung der Frage der zulaessigen Anstrengung der Konstruktions-materialien", von G. D. Sandel, Leipzig, 1925.

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DISCUSSIONS

THE STRESS FUNCTION AND PHOTO-ELASTICITY APPLIED TO DAMS

Discussion

BY FRED L. PLUMMER, ASSOC. M. AM. SOC. C. E.

FRED L. PLUMMER,[25] ASSOC. M. AM. SOC. C. E. (by letter).[25a]—It is unfortunate that so few practicing engineers are able to make use of many of the more fundamental and more generalized stress functions. In illustrating so effectively one application of the Airy stress function the author has performed a difficult but exceptionally valuable service to the profession. The writer wishes to discuss Part II of the paper which describes the use of photo-elastic studies in determining the distribution of principal and shearing stresses in dams. This method of analysis has many proper applications in the field of structural engineering and should be more generally understood and used by designing engineers. In many cases, quite crude apparatus will give qualitative results of great value.

The author calls attention to the difficulties encountered in attempting to simulate body forces (due to weight and inertia) in the model. The writer has met this problem in two ways. In one study a model was rotated together with a slotted wheel giving a stroboscopic effect. This method is similar to that suggested in the paper and gave satisfactory results. The apparatus is somewhat elaborate, however, and requires careful adjustment.

A much more direct solution may be effected by the use of models made of gelatin. This material may be secured in either "ground" or "sheet" form, at little cost. It should be soaked in cold water for at least 1 hr and then heated to a temperature of not more than 150° F. The concentration of the solution can be varied from 2% or 3% to about 50% (by weight), giving a wide range to the strength and stiffness of the mass which solidifies upon cooling. The stress-strain-optical properties are such as to make the material quite suitable for photo-elastic studies when body forces are of importance.

NOTE.—The paper by John H. A. Brahtz, Esq., was published in September, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by I. K. Silverman, Jun. Am. Soc. C. E.

[25] Assoc. Prof., Structural Eng., Case School of Applied Science, Cleveland, Ohio.

[25a] Received by the Secretary November 8, 1935.

These properties vary, however, as the material ages, and it is necessary, therefore, to make calibration tests at the time of the model tests. The writer has found concentrations of from 10 to 20% most suitable for models of gravity dams. Such models may have thicknesses as great as 6 in., as compared with a fraction of an inch which is common for models made of bakelite. This fact, together with the greater stress-optical sensitivity of the material, makes easy the determination of the stresses caused by the weight of the material itself. The use of much larger models is also made practical.

The writer has made several studies using this type of model. A brief description of one such study conducted in co-operation with the Zanesville, Ohio, Office of the U. S. Corps of Engineers may be of general interest. Major J. D. Arthur, Corps of Engineers, U. S. Army, District Engineer, T. T. Kappen, Assoc. M. Am. Soc. C. E., Chief of the Engineering Division, A. L. Alin, M. Am. Soc. C. E., Chief of the Dams and Reservoirs Division, and R. R. Philippe, Director of the Soils Laboratory, were responsible for the planning and conduct of the tests.

The structure under consideration was a gravity type of concrete dam. A typical cross-section of the spillway section is shown in Fig. 14. Two series of photo-elastic model studies were made. Bakelite models were used in the first series, the procedure being very similar to that outlined by the author, except that vertical loads were applied so as to simulate, successively, the proper intensity of weight forces at three different elevations (Elevation 850, Elevation 870, and Elevation 890). Gelatin models constructed to a scale of 1:60 were used for the remaining studies. The weight of the prototype was represented accurately by the weight of the model while the forces due to water pressure were reproduced by the use of a liquid having a weight, in pounds per cubic foot, bearing the same relation to 62.4 as that of the unit weight of the gelatin as tested had to the assumed unit weight of the concrete. Liquid pressures were applied directly to the model through a thin rubber diaphragm. It is believed that the external forces as well as the body forces were thus represented accurately. It was possible to study the changes in the combined stresses as the water level might rise in the prototype from empty reservoir to maximum flood level estimated in this case as Elevation 931, which corresponds to 15 ft of water over the top of the spillway. By varying the relative strength and stiffness of the mass of gelatin representing the foundation material, it is possible to study the effect of foundation materials of varying strength characteristics upon the stresses near the base of the dam.

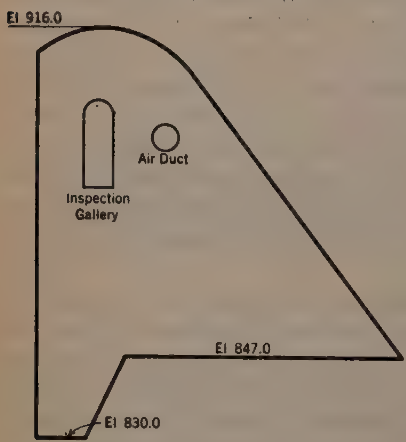

FIG. 14.

In this case it was especially desired to determine the stresses in the material around the openings for the air duct and inspection gallery, and in the cut-off anchor wall which was provided because of horizontal layers of soapstone that occurred in the foundation material. The results of the tests were highly satisfactory, the final design of steel reinforcement at these points being based on the test results.

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DISCUSSIONS

FLOOD AND EROSION CONTROL PROBLEMS AND THEIR SOLUTION

Discussion

By C. S. Jarvis, M. Am. Soc. C. E.

C. S. Jarvis,[13] M. Am. Soc. C. E. (by letter).[13a]—Although the problem outlined in the paper may be more nearly representative of extreme conditions than of average or usual flood and erosion control problems to be encountered in the United States, it follows that an adequate solution for the more difficult situation may be applicable, at least in part, for the simpler ones.

The observed and reasonably expected rates of precipitation for the foot-hill region around Los Angeles, Calif., are far from the maxima observed elsewhere for both long and short periods, particularly along or near the Gulf Coast. Likewise, the rates of run-off per square mile of drainage area thus far observed in that foot-hill region are considerably less than have been recorded for numerous small streams. The outstanding or unique features of the situation described by the author comprise the following: (1) Steep slopes on the windward side of relatively high mountain ranges, and near the coast; (2) warm or moderate all-year temperatures except on the higher elevations; (3) seasonal precipitation occurring mainly during the winter; (4) droughts occurring every summer, thus limiting the types of surviving vegetation and resulting in recurrent fire hazards; (5) unconsolidated débris and soil materials depending for their stability, on the steeper slopes, largely on the roots of vegetation; (6) soil surface partly protected by plant foliage, and incapable of maintaining itself without such protection; (7) metropolitan and urban developments on the detrital cones and flood-plains; (8) local water resources deficient, especially during the seasons of maximum demand; (9) justification for storage developments to utilize all the normal run-off, thence to facilitate recovery and repeated use; (10) ground-water supplies heavily overdrawn, with consequent intrusion of salt water to menace and further deplete the remaining ground-water reserves; (11) unusual opportuni-

Note.—The paper by E. Courtland Eaton, M. Am. Soc. C. E., was published in September, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by Messrs Arthur G. Pickett, and R. W. Davenport.

[13] Hydr. Engr., Soil Conservation Service, Washington, D. C.

[13a] Received by the Secretary November 6, 1935.

ties for flood regulation by spreading, and by storage in basins partly bounded by loose, permeable strata, for the double purpose of local protection of lives and property and prompt replenishment of underground storage; and, (12) property values capable of supporting a comprehensive program for flood regulation and protection, water conservation, and its maximum utilization.

It is readily granted that other localities may be characterized by one or more of the aforementioned features which occur in combination in the Los Angeles District; but nowhere else in this country are all the factors represented to such a degree. Therefore, the measures economically justified for the Los Angeles situation may be applicable only in part elsewhere.

It has been widely observed among the head-waters of streams in both foot-hill and mountainous regions that minor landslides are going on progressively during any normally wet season, and occasionally during periods of deficient rainfall. The planes of contact between ledge rock and products of weathering are usually planes of weakness and instability; this is also true of rock débris on inclined beds of clay or other relatively impervious material. Water trickling along such planes naturally acts as a lubricant, inducing motion, cleavage, subsidence, and occasionally landslides.

Such surface disturbances among head-waters are so common and so unimportant as to escape prominence. The remoteness from the main channel of the stream, the relatively small volumes of water yielded from the disturbed areas, and the correspondingly small amount of erosion resulting therefrom, all combine to keep such phenomena in the background. However, when they occur along the lower courses of streams, where lives are endangered and property threatened, they assume considerable prominence. It appears that some of the unstable masses on such slopes are susceptible of treatment to insure their permanency in present positions; or, occasionally, to accelerate their stabilization by inducing movement to flatten the slope to a natural grade of repose, corresponding to most severe conditions expected to be encountered.

A parallel is afforded in the treatment of sites for flumes, pipe lines, or other structures near the foot of a slope. It is advisable and customary to clear the upper slopes of boulders or other rock masses that might be loosened or detached by weathering processes; or, occasionally, it is deemed expedient to repair or connect protruding ledges to serve as barriers for the protection of the lower slope.

Recent practice in the construction of trails and access roads among scenic mountain areas deserves more than passing notice. To prevent accumulations of water on the trails or roadways, diverting channels have been constructed to discharge into basin-like depressions, such as the borrow-pits from which surfacing material has been obtained. In such cases, where several feet of depth and potential hydrostatic pressure are provided, and where coarse materials, such as gravel or rock waste and other loose materials are exposed, the rapid percolation and the storage combine to dispose of surface run-off. The ease with which a large number of such percolation and storage facilities may be provided seems to make up for their restricted capacity, taken separately.

Several municipal water supply or irrigation projects, as well as hydro-electric power projects, have proved failures due largely to leakage from the reservoirs. Where comprehensive developments are undertaken for the benefit of all legitimate interests on a river system, it may well prove to be a profitable venture to seek reservoir sites that will feed underground storage by leakage or percolation. Where test borings disclose coarse, pervious materials at reasonable depths below the valley floor, it may be advisable to trench or cut through the more nearly impervious overlying material to induce underground flow and storage.

Modifications or adaptations of the foregoing methods may well be adopted in connection with flood and erosion control problems. Particularly, the provision for many avenues of access to strata of coarse, open texture, and multiple small storage basins for both water and débris may measurably relieve the burden to be sustained by the larger reservoirs down stream, helping to maintain their original capacity and thus adding to their effectiveness.

Doubtless, general recognition of existing facts would prove that nearly every locality is confronted with flood and erosion control problems, each meriting individual treatment and capable of a best solution; or, where misguided by inexperienced or over-confident advisers, a community may appropriate and spend generously without measurable progress toward a satisfactory solution. The author has done much to clarify the view as to potential dangers from those natural phenomena, and as to remedial steps which have proved effective in the Los Angeles District. Mr. Eaton has rendered a distinct service by his able presentation of the subject-matter.

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DISCUSSIONS

HYDRAULIC LABORATORY RESULTS AND THEIR VERIFICATION IN NATURE

Discussion

By Messrs. Samuel Shulits, and Herbert D. Vogel

Samuel Shulits,[21] Jun. Am. Soc. C. E. (by letter).[21a]—In a convincing manner, Captain Vogel presents the practical value and qualitative reliability of river models: However, it is the problem of the quantitative evaluation of fluvial models that confronts laboratories everywhere to-day and further systematic research is necessary to clarify this phase of river-model usefulness. At present, the writer is inclined to agree with Koerner[22] that the results of properly planned experiments with movable-bed models are always qualitatively transferable to the prototype in Nature. In other words, it is always determinable which of the investigated construction measures promises the greatest efficacy in the desired direction. According to Koerner, it is still necessary, however, to exercise great care in applying quantitative conclusions derived from river-model studies to predict the behavior of prototypes.

Although the path may be difficult, there is gratifying evidence that movable-bed models can yield quantitative results. For example, the Schoklitsch bed-load formula[23] was developed rationally, its constants were determined in a laboratory flume, and the results have been found to hold for several European rivers. It correlates bed load, size of material moved, energy gradient, and discharge.

Furthermore, it has actually been possible to find an accurate quantitative relation between the discharge, the head, the size of the bed material, and

Note.—The paper by Herbert D. Vogel, Assoc. M. Am. Soc. C. E., was published in January, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1935, by Messrs. W. F. Heavey, Chilton A. Wright, Paul S. Reinecke, Morrough P. O'Brien, and John A. Jameson, Jr.; and August, 1935, by Messrs. J. C. Stevens, and Paul W. Thompson.

[21] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[21a] Received by the Secretary October 31, 1935.

[22] "Modellversuche für einen Fluss mit starker Geschiebebewegung ohne erkennbare Bankwanderung", by H. D. Krey, prepared and edited by Burghard Koerner, Berlin, 1935, p. 38.

[23] "Der Geschiebetrieb und die Geschiebefracht", by A. Schoklitsch, *Wasserkraft und Wasserwirtschaft*, No. 4, 1934, p. 37; also "The Schoklitsch Bed-Load Formula" by Samuel Shulits, *Engineering*, June 21 and 28, 1935, pp. 644 and 687.

the pool depth below a jet falling freely over a sharp-crested weir[24] in a small model. In this case, eight different sands of non-uniform composition were used. To be sure, some may say that the relation thus obtained is valid only for the small model, but it must be realized that the search for even such a relationship was considered futile not long ago.

Lack of agreement between siphons and their models has been reported by A. Veronese.[25] Three models, of different scales, were made of siphons

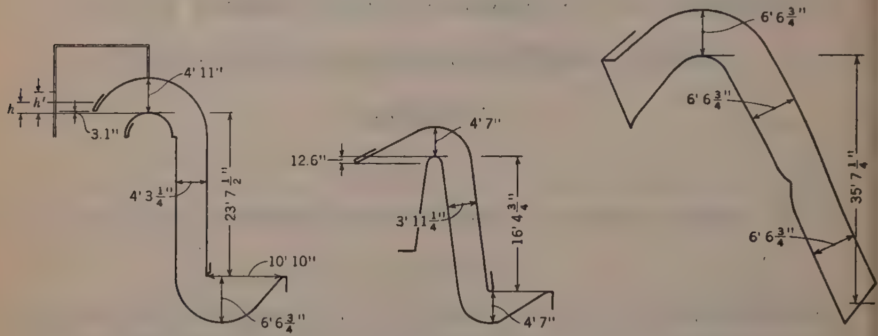

FIG. 19.—SIPHONS INVESTIGATED BY ALESSANDRO VERONESE.

(Fig. 19) in Italy; the purpose of the tests was to determine the relationship between the minimum head (h, in Fig. 19), and the corresponding time, T, required to prime, in both model and prototype. If the subscripts, m and n, denote, respectively, model and prototype, and if the scale ratio is expressed as $1:a$, then, with Froude's law as a basis, the relations between priming head and priming time should be:

$$h_n = a\, h_m \qquad (4)$$

and,

$$T_n = T_m \sqrt{a} \qquad (5)$$

but Veronese's experiments led him to conclude that Equations (4) and (5) are invalid, and he derived the following from the data:

$$h_n = h_m\, a^b \qquad (6)$$

and,

$$T_n = \frac{T_m}{\sqrt{a}} \qquad (7)$$

in which $b = \dfrac{24 \sqrt{a}}{T_m}$. The agreement (which can be termed sufficient, per-

[24] "Kolkbildung unter Ueberfallstrahlen", by A. Schoklitsch, *Die Wasserwirtschaft*, 1932, No. 24.

[25] "Ricerche sulla relazione che intercede tra l'altezza di adescamento dei sifoni autolivellatori sperimentati in modello e quella dell'originale", by Alessandro Veronese, *L'Energia Elettrica*, July, 1934, p. 517.

TABLE 3.—COMPARISON OF OBSERVATIONS AND COMPUTATIONS BY VERONESE'S EQUATIONS

Siphon	Model scale, a	MINIMUM PRIMING HEAD, h, IN INCHES			PRIMING TIME, T, IN SECONDS		
		Observed	Prototype Computations		Observed	Prototype Computations	
			By Froude's law	By Equation (6)		By Froude's law	By Equation (7)
Camuzzoni (Fig. 19(*a*))	20	1.04	20.80	6.97	170	760	38.03
	10	1.11	11.10	4.72	120	379	37.97
	5	1.33	6.65	3.90	80	179	35.71
	1	5.12	5.12	5.12	....	....	....
Carron (Fig. 19(*b*))	20	2.90	58.00	9.61	270	1 208	60.40
	12	2.88	34.55	8.03	210	727	60.69
	8	2.93	23.42	6.73	170	481	60.07
	1	7.20	7.20	7.20	60	60	60
S. Caterina (Fig. 19(*c*))	25	1.90	47.50	19.87	165	825	33.00
	15	1.95	29.23	15.70	120	465	31.00
	8	2.72	21.76	12.91	90	255	31.80
	1	15.75	15.75	15.75	....	....	....

haps, although not close) between the observations and values computed with Veronese's equations is shown in Table 3. The usual equations based on Froude's law do not give as close an agreement. The prototype priming time was measured only on the Carron siphon.

HERBERT D. VOGEL,[26] ASSOC. M. AM. SOC. C. E. (by letter).[26a]—In 1929 there was not a single hydraulic structures laboratory in the United States, in a strict sense, although plans were being considered for eight to be established at various places.[27]

A recent *Bulletin*[28] of the National Bureau of Standards contains descriptions of fifty-three American laboratories, twenty-two of which are shown to be operated largely for the solution of open-channel problems. This phenomenal development may be due partly to the great increase in public work activities during the past few years, but inference is strong that some credence should be given the explanation that model studies return ample dividends. It should follow as a corollary, then, that small scale models may be depended upon for the procurement of trustworthy information.

In an effort to find proof—or disproof— of the reliability of open-channel experimentation, this paper was tossed out as a challenge. The results, while disappointing, were nevertheless illuminating in their paucity, and the question could now be asked, "Are current studies being undertaken: (1) Because of the proved validity of the method; (2) because models are a cheap means of approximation; or (3) simply because it is fashionable to carry on 'scientific' research?" To run down the answer to this query circular letters were sent out from the U. S. Waterways Experiment Station during August, 1934, to more than fifty laboratories or individuals engaged in hydraulic research.

[26] Captain, Corps of Engrs., U. S. A., The Command and General Staff School, Fort Leavenworth, Kansas.

[26a] Received by the Secretary November 13, 1935.

[27] "Hydraulic Laboratory Practice". Edited by the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., p. 731, Am. Soc. Mech. Engrs., N. Y., 1929.

[28] "Hydraulic Laboratories of the United States", *Hydraulic Laboratory Bulletin, Series B*, National Bureau of Standards, First Revision, October 1, 1935.

It was requested in each case that information be furnished regarding the performance of any model tests, the results of which had been substantiated subsequently by results obtained in the field. Twenty of these letters were directed to addresses within the United States and the remainder were sent to foreign countries.

Replies were received from twenty-eight of those addressed, twelve of these being from laboratories within the United States. With a few notable exceptions, the replies contributed scarcely any information directly pertinent to the subject. From Germany came the citation of one instance of a model (scale, 1:45) of a dam that had produced results which, when compared with recorded data of the completed structure, were found to be accurate within 10 per cent. One correspondent in Italy reported that tests on siphons had failed to confirm the laws of similitude, whereas another stated that models of siphon spillways had been tested and found to reveal true information as to what could be expected in Nature.

A comparative wealth of data was received from Holland, outstanding among which were the following:

(*a*) Model studies to determine the efficiencies of two centrifugal pumps gave values varying from 0.75 to 0.87. In the prototype the efficiencies were found to be 0.80 in each case.

(*b*) After improving the Harbor of Breskens the intensity of wave action was reduced somewhat, as had been indicated by a model study.

(*c*) The results of six experiments with models (scale, 1:20) to determine the movement of stone on fascine mattresses were found to be in close agreement with the results obtained later in the Maas River.

(*d*) A 1:50 model of a discharging sluice gave depths of scour that were almost exactly verified in the field.

(*e*) A model (scale, 1:25) was operated to determine the discharge capacity of a sluice. Exact comparison with the data derived from full-scale performance has not been possible, but there appears to be a close agreement.

(*f*) A 1:5 scale model was built to study the flow of viscous fluids in suction pipes. Prototype and model flows were found to be exactly similar.

(*g*) Six models of navigation locks were experimented with in an effort to determine times of filling and forces acting upon the ships during lockage. When these locks had been built to full-scale it was found that for two of them the times of filling were within 10% of those figures indicated by the model tests. Exact comparisons were not possible in the other cases, but it appeared that the general results were as had been predicted. Some indications of the produced forces were obtained from the full-scale structures, from which it appeared that differences of 15 to 25% were about the least that could be expected.

Sweden supplied information of two model tests which were subsequently verified by performances in full scale. One of these was to determine the degree of erosion that might be expected below the sector gates of a power plant; the other pertained to the procurement of data on a regulating weir.

Verification was received from England on the performance of tidal models; and data of surge tank experiments were also presented, together with those obtained later in the field. The results indicated an almost exact agreement between surge heights in model and in Nature, and revealed close agreement in periods.

Three of the twelve American laboratories that replied gave verifications of data obtained from model spillways, locks, and dams. Little or no information was received in justification of movable-bed models.

There can be no doubt that field verifications of small-scale experiments are difficult to obtain, but if models are to continue in their present high place as auxiliary aids for the hydraulic engineer, it will be necessary for their sponsors to procure positive proof of their reliability. Furthermore, it is just as important that the failures be revealed and recorded because, unless this is done, undue reliance may be placed on the results of certain studies from time to time. The scientist and the engineer must face facts as they are, prepared to modify their convictions as the need for so doing is evidenced. The several discussions of the paper carry this thought as an underlying consideration. The time for proselyting is past; careful scrutiny and study are now in order as means of determining what types of models can be relied on for quantitative data; what kinds will give reasonably accurate qualitative effects; and which are capable of nothing but deceit.

An extremely common sense viewpoint of the entire question has been presented[29] editorially. One paragraph appearing particularly worthy of quotation is as follows:

"The publication of results, with particular reference to any new form of technique evolved, is greatly to be encouraged, in all hydraulic model work, no less, of course, than in any other branch of applied mechanics; for the guidance so provided may serve not only to help others to avoid the many pitfalls, but may effectively reduce the cost and labour involved. Certainly the cost of such investigations cannot be lightly undertaken, the labour entailed both in the laboratory and often also in the very important work of collecting essential data for the experimental work from the field, is considerable. Speaking generally, however, in cases where the need for model investigations is felt, the cost will be small in comparison with the scheme in hand."

This comment was written with the writer's paper in mind.

[29] *Engineering* (London), May 31, 1935.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE ADJUSTMENT OF A LEVEL NET

Discussion

By George H. Dell Assoc. M. Am. Soc. C. E.

George H. Dell,[9] Assoc. M. Am. Soc. C. E. (by letter).[9a]—The discussions submitted on this subject by Professors Church and Rayner contain interesting and helpful comments. Both writers find the distribution method superior from the standpoints of simplicity and applicability.

The example analyzed by Professor Rayner is a typical indication of the errors inherent in "compromise" methods of adjustment. Professor Church has clarified an important item in pointing out that the corrections are substantially independent of the order of adjusting the separate circuits. He has also made noteworthy observations in regard to the elimination of undue refinements of precision.

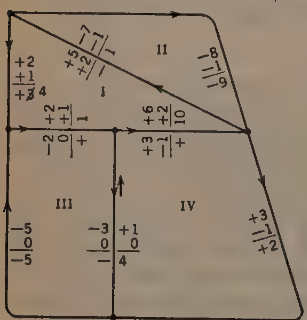

Fig. 6.—Simplified Adjustment of Level Net, Example No. 1

The original solutions were prepared with a view to presenting methods capable of yielding results correct to approximately 0.001 ft, and therefore, applicable to precise surveys. For ordinary level nets, in which an inaccuracy of 0.01 ft in any section is permissible, two cycles will generally be sufficient if supplemented by occasional arbitrary corrections of 0.01 ft. Fig. 6 illustrates the solution of Example No. 1 under such conditions. It will be noted that at the end of the second cycle the only remaining error was in Circuit No. I; this was eliminated by changing the correction of Section No. 1 to + 0.04 ft.

Note.—The paper by George H. Dell, Assoc. M. Am. Soc. C. E., was published in April, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1935, by Messrs. Earl F. Church, and W. H. Rayner.

[9] Associate in Civ. Eng., Univ. of Illinois, Urbana, Ill.

[9a] Received by the Secretary November 4, 1935.

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DISCUSSIONS

FLOOD-STAGE RECORDS OF THE RIVER NILE

Discussion

By Messrs. R. W. Davenport, H. E. Hurst, Thomas H. Means, J. W. Beardsley, and J. C. Stevens

R. W. Davenport,[27] M. Am. Soc. C. E. (by letter).[27a]—The occurrence of a flood is determined by the conjunctive operation of a variety of meteorologic, hydrologic, topographic, and other factors and conditions in a manner favorable to the production of abnormally excessive flows. As knowledge of these fundamental factors and conditions has become more plentiful through the collection of records at rainfall and river measurement stations, topographic surveys, etc., it has been possible to introduce more science into the analysis of flood occurrence and thereby to advance somewhat in relating floods to their constituent elements. Notable advances of this kind have been made in recent years by different individuals and organizations along more or less closely related lines, with resulting improvement in the technique of flood study and the promise of further improvement as the supply of essential base data increases and the knowledge of practical and theoretical hydrology progresses.

The flood record of the Nile River, as presented in the paper by Mr. Jarvis, is the result of a thorough and painstaking endeavor to reconcile conflicting evidence of Nile flood heights as derived from various sources and to convert the information into a form that can be readily understood and discussed. Future students of the long Nile record may have the advantage of starting from the advance point to which Mr. Jarvis has progressed in his researches. Although numerous uncertainties and difficulties tend to place the record somewhat in a cloud of doubt as regards its reliability of detail, full knowledge of the facts as developed will be of great value to future investigators. The writer believes with Mr. Jarvis that despite these unfortunate defects of the record the Engineering Profession may be able to derive from it valuable knowledge in regard to long-time characteristics of river behavior, and particularly in regard to the changes in behavior with respect to floods within the centuries covered by the record.

Note.—The paper by C. S. Jarvis, M. Am. Soc. C. E., was published in August, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by Halbert P. Gillette, M. Am. Soc. C. E.

[27] Hydr. Engr. in Chg., Div. of Water Utilization, U. S. Geological Survey, Washington, D. C.

[27a] Received by the Secretary November 1, 1935.

The proposition that the characteristics of flood behavior of a stream may be best understood and dealt with by analyzing them in relation to their fundamental constituent elements has the direct corollary that such an analysis of these characteristics for a certain stream may also best contribute to knowledge of corresponding characteristics for other streams, neighboring or distant. Obviously, therefore, if the flood characteristics of the Nile and other foreign rivers, as displayed by long records, are to be made rationally applicable to streams elsewhere in the world they require to be analyzed and translated, so far as practicable, into common fundamental terms. In undertaking to determine, in a superficial way at least, something of the significance of the Nile record in relation to other streams, the writer has been interested to review some of the outstanding features of the Nile floods.

The Nile drainage basin has an area of 1 119 737 sq miles, and the river has a waterway length of 3 728 miles.[28] The corresponding values for the Mississippi-Missouri System are 1 240 000 sq miles and about 4 200 miles.

The main Nile is formed by the junction of the White Nile and the Blue Nile at Khartoum, 1 913 miles above the mouth. Through this 1 913-mile reach below the junction the river flows through a very arid country and except for the River Atbara, which enters from the east, 159 miles below the junction, the tributary area is small and the inflow practically negligible. The flow is subject to substantial diminution by evaporation and seepage; moreover, for thousands of years each flood has supplied substantial volumes of water, largely from the lower part of this reach, for the irrigation of the fertile Nile Valley. The flow of the river is derived from two main sources: (1) The lakes and great swamp areas of Central Africa drained by the White Nile and its tributaries, with a heavy tropical rainfall distributed through the greater part of the year; and (2) the highlands of Ethiopia, drained by the River Sobat, tributary to the White Nile from the East, the Blue Nile, and the River Atbara—a region marked by an extraordinary concentration of rainfall and run-off in the period from July to September. Thus "reduced to its simplest expression, the Nile system may be said to consist of a great steady-flowing river fed by the rains of the tropics, controlled by the existence of a vast head reservoir, and annually flooded by the accession of a great body of water with which its eastern tributaries are flushed."[29]

TABLE 3.—DRAINAGE AREAS OF ETHIOPIAN EFFLUENTS

River	Drainage area, in square miles	Distance of junction from mouth of the Nile, in miles
Sobat	94 560	2 429
Blue Nile	127 998	1 913
Atbara	85 216	1 754
Total	307 774	......

[28] These and other statistical data of the Nile presented herein are obtained from "The Physiography of the River Nile and Its Basin", by Capt. H. G. Lyons, Director Gen., Survey Dept., Finance Ministry, Egypt, Cairo, National Printing Dept., 1906.

[29] "Encyclopædia Britannica, 14th Edition, Vol. 16, p. 454.

Table 3 shows, in down-stream order, the drainage areas of the Ethiopian effluents that are the sources of the annual Nile flood and the relative locations of their junctions with the main river. The Blue Nile rises above Lake Tsana, about 2 800 miles from the mouth of the Nile.

Some understanding of the seasonal distribution of the rainfall that is the source of the Nile floods may be obtained from Table 4, which gives the mean monthly rainfall at Addis Ababa for a period of 13 yr and 2 months.[30]

The hydrographs of flow at stations in the upper reaches of the tributaries[31] show a most pronounced flood rise extending from July to the late autumn and culminating in a maximum stage in August or September (averaging about September 1). The hydrographs at the upper stations follow an irregular saw-tooth line, thus displaying direct sensitiveness to the variations of rainfall distribution over the upper part of the drainage basin.

TABLE 4.—MEAN MONTHLY RAINFALL AT ADDIS ABABA, ETHIOPIA

Units	January	February	March	April	May	June	July	August	September	October	November	December	Total
Millimeters.....	15	39	70	82	64	142	283	302	163	16	19	3	1 198
Inches..........	0.59	1.54	2.76	3.23	2.52	5.60	11.13	11.90	6.42	0.63	0.75	0.12	47.19

The flow of the White Nile above the mouth of the Sobat is extraordinarily regular. The influx of the Sobat tends to swell the flow materially, beginning with June, but apparently the river continues to be characterized by unusual regularity until it approaches the mouth of the Blue Nile, where it shows greater domination by the influences that create the Nile floods. Below the junction of the Blue Nile, the flow assumes the definite pattern of the Nile floods.

Farther down the main river the hydrographs become continuously more smooth, until at the Roda gauge, at Cairo, where the long record has been kept, the incisions in the hydrograph that are due to natural causes are undoubtedly very small. Moreover, under natural conditions, the time of maximum stage at Cairo is about October 1 on the average, or about a month later than on the upper reaches of the tributaries.

From the available information it is apparent that:

(1) The rainfall that creates the Nile floods is concentrated largely in a 90-day period;

(2) The collecting system of the flood waters integrates the variable flows originating from the heavy rainfall over a wide area in a peculiarly effective manner, so that the flood at the place where it enters upon its long course to the sea through the main Nile channel is characterized by a remarkable uniformity of pattern from year to year; and,

[30] "The Rains of the Nile Basin and the Nile Flood of 1912", by J. I. Craig, Director, Meteorological Service, Survey Dept., *Survey Department Paper 32*, Ministry of Finance, Egypt.

[31] As pub. in the *Survey Department Papers* of the Ministry of Finance, Egypt.

(3) The passage through about 1 900 miles of capacious flood channel, with the resultant regulating influence of a great volume of channel storage, still further smooths the hydrograph of flow, except as artificial regulation and diversion have produced irregularities, usually of minor character.

The only drainage basin in the United States that is comparable in area and length to that of the Nile is the Mississippi Basin, but its hydrologic characteristics are quite dissimilar. The Colorado Basin is similar to the Nile Basin in respect to the fact that its flood is peculiarly seasonal in its occurrence, but it is much smaller and quite unlike the Nile Basin in various other ways. It is evident, therefore, that the application of the Nile experience to the rivers of this country is not direct and easy, but requires an appropriate analytical treatment of fundamental factors.

In reviewing the evidence of past centuries, therefore, it appears that the annual maximum stage at the Roda gauge represents the culminating height attained by a flood rise of large volume and unusual regularity of pattern. Outstanding features of the flood are its regular seasonal recurrence and its obscuring of erratic tendencies, both in distribution of rainfall and in the characteristics of drainage basins in shedding flood waters. Any application of the facts disclosed by the Nile record to streams elsewhere, of course, should take these and other essential characteristics appropriately into account.

H. E. Hurst,[32] Esq. (by letter).[32a]—The Roda nilometer situated in Cairo is said to be the oldest existing Arab monument in Egypt, and consists of a well of about 5 m (16.4 ft) square communicating by tunnels with the river and having a vertical column at its center on which the scale is cut. The column is held at the top by a cross-piece of masonry spanning the well, and the water levels were read from a staircase that winds around the outer edge. Its records dating back to A. D. 622, although not complete, form perhaps the longest written record of any meteorological phenomenon.

The originals as far as known do not exist at present, and the form in which the records were kept is also unknown. Colonel Sir Henry Lyons who came to Egypt in the Nineties was told that the original records were written in Coptic and existed in the Ministry of Public Works in the time of Aly Pasha Mobarak, a few years before his arrival in the country, but he was never able to trace them. The writer has never heard of a record with any claims to antiquity, and if any such existed in Egypt, H. H. Prince Omar Toussoun who has collected so much old material about the Nile would probably have discovered it.

In assessing the value of these records many points need consideration (some of which will be mentioned subsequently), and the investigator must divest himself of the idea that they are comparable in accuracy with present-day records. The scientific spirit of recording exactly what is observed, without personal bias and with every effort to eliminate sources of error, is a recent growth and is not to be expected in records the most complete portion of which is from 600 to 1400 A. D.

[32] Director-General, Physical Dept., Ministry of Public Works, Cairo, Egypt.

[32a] Received by the Secretary November 7, 1935.

The Roda nilometer has been repaired on various occasions recorded by historians, in some of which the repairs have been so extensive as to leave a doubt as to whether it was not entirely reconstructed. There is little doubt that the list of repairs is incomplete and in no case is there any account of what steps, if any, were taken to preserve the datum level during the repairs.

Apart from changes of datum of an accidental nature one must consider the effects of changing topography and varying regime of the river such as that which is known to have occurred in historical times. An example of this mentioned in the paper is the varying number and position of channels through the Delta, and a further statement that the position of the western bank of the river near Cairo has changed by nearly 1 km (0.6 miles) since Napoleon's time.

A point that may be mentioned is the effect of the basin system of irrigation on the river levels at Cairo. In this system, which was the only one practised in Egypt until the Nineteenth Century, the country is divided into sections by banks transverse to the course of the river, and the flood waters of the Nile are admitted to these sections or basins by means of short canals. After a period of forty days or more the water is allowed to escape back to the river and the basins are planted with crops. In recent times, basin irrigation has been well controlled by means of masonry regulators on the canals and escapes, and the water is admitted to and escaped from the basins according to a program. The effect of this is to alter the natural flow of the river at Cairo very considerably, lowering the natural peak and the succeeding levels to raise them again when water begins to escape. In some cases the second peak has been higher than the first one and, in others, it has not been noticeable, the actual regime depending largely on the nature of the flood. Since 1900, the basin area of Upper Egypt has been diminishing gradually, owing to the conversion of the basins to the perennial system.

Previous to the Nineteenth Century basin irrigation must have been a much more haphazard process, as lack of communications would prevent its regulation on an organized plan directed from headquarters. Moreover, there would be no masonry regulators; the only control of entrance to, and escape from, basins would be by earthen banks; and after these were once cut the water would take its course. Possibly the water merely flowed through breaches in the river bank as the flood rose, and back through the same breeches as the river fell, making the process more or less a natural one, which would be modified occasionally by the bursting of cross-banks. In very early times the cross-banks probably did not exist, and the annual inundation of Upper Egypt was a still simpler phenomenon.

Unfortunately, no details of the form of the Roda gauge curve previous to the Nineteenth Century have been handed down, and only readings purporting to be the maximum and minimum, are available. The minimum readings would be much more likely to be affected by changes of topography, such, for example, as the closing of a main channel near Cairo by a silt bar or an earthen bank (as must frequently have happened), or by the gauge itself being shut off from the main stream.

The possible exaggeration of low floods for revenue purposes has been mentioned in the paper. Another source of error is that the gauge pillar must always have been difficult to read as the observer cannot approach it closely; furthermore, the scales became worn and often no doubt were incrusted with mud. This led to a practice which was followed by some of the gauge readers of recent times and was referred to by Col. J. C. Ardagh in 1889,[33] of reading not on the gauge, but on private marks either on the wall of the gauge-well or on a flight of steps outside, leading down to the Nile.

Some of the sources of error that affect the records are of an accidental nature and will be largely eliminated by the number of observations, which is approximately 1 100 both of flood stage and of low stage.

Systematic changes such as the gradual rise of the river bed can be eliminated for some purposes by dividing the observations into groups of, say, 100 yr and dealing with the departures from the mean of the century. This procedure helps also to eliminate the effects of topographical changes. Deliberate exaggeration of low floods cannot be eliminated, but it is quite likely that it is a practice which was only used for a time and does not affect all the records, and, therefore, its effect is minimized when the entire mass of material is discussed.

As a whole, these records represent observations extending over a very long period of the rise and fall of the Nile. They have not the precision of modern observations, but are probably as reliable as many of the statistics collected at the present day about less well-defined phenomena, such as the health and social conditions of mankind.

Two uses to which these records have been put in recent times are: (1) Analysis to discover the existence of periodicities; and (2) construction of a curve giving the frequency of floods of different heights.

The old records of the Roda gauge given by various authors were scrutinized by Mr. J. I. Craig, of the Egyptian Ministry of Finance, and converted to a metric scale, and it is this collection, with certain minor changes, that has been used in the Physical Department, Egypt, or communicated by the Department to inquirers.

Periodic analyses have been made by the late Professor H. H. Turner[34], and by Messrs. J. I. Craig, and C. E. P. Brooks[35], and periods varying from 2 to 240 yr have been found. The period of greatest amplitude so far is one found by Professor Turner of 240 yr, with an amplitude of 15 cm (6 in.) for the maxima and 46 cm (18 in.) for the minima.

The analysis by Mr. Brooks does not extend beyond a period of 76.8 yr, but he finds a number of periods of average amplitudes of the order of 10 cm (4 in.). His best defined periodicity is the one of 76.8 yr, with a mean amplitude of 17 cm ($6\frac{5}{8}$ in.). The average standard deviation of the flood levels is 56 cm (22 in.); this makes apparent the relative smallness of any periodic

[33] "Nilometers", by Col. J. C. Ardagh, *Proceedings*, Royal Geographical Soc., Vol. XI, No. 1, January, 1889.

[34] "On a Long Period (240 Years) in Chinese Earthquake Records", by H. H. Turner, Monthly Notices, Royal Astronomical Soc., May, 1919, Vol. LXXIX.

[35] "Periodicities in Nile Floods", by C. E. P. Brooks, *Memoirs*, Royal Meteorological Soc., Vol. II, No. 12.

effects, which although of theoretical interest, are of no use to the forecaster. A glance at the records when plotted on a fairly large scale shows that there is no period which is directly evident to the eye, and that the principal features are the existence of fairly long terms of years when, on the whole, the floods have been high and others when floods have been low. This fact is well illustrated by the Aswan gauge records for the period, 1869–1933. By dividing this period into two parts, 1869–1898 and 1899–1933, an example is afforded of a high term of 30 yr followed by a low term of 35 yr, with the results indicated by Table 5.

TABLE 5.—RECORDS OF ASWAN GAUGE DIVIDED INTO TWO PERIODS

Description	1869–1898			1899–1933		
	In meters	In feet	Maximum year	In meters	In feet	Maximum year
Maximum gauge reading of highest flood	94.15	308.9	1878	93.30	306.1	1908
Maximum gauge reading of lowest flood	91.40	300.9	1877	90.11	295.6	1913
Mean flood maximum for the period	93.26	306.0	....	92.39	303.1	....
Number of years with maxima greater than 93.30 m.	15			0		
Number of years with maxima less than 91.40 m.	0			2		

The most striking feature of Table 5 is that in the high term, one year in every two was higher than the highest flood of the following low term. No regularity in the occurrence of these high and low terms has been discovered so far; nor are floods uniformly high in a high term or low in a low term.

A frequency curve was constructed by the writer from all available Roda gauge maxima. This curve was used to estimate the frequency of occurrence of very low floods such as that of 1913.[36]

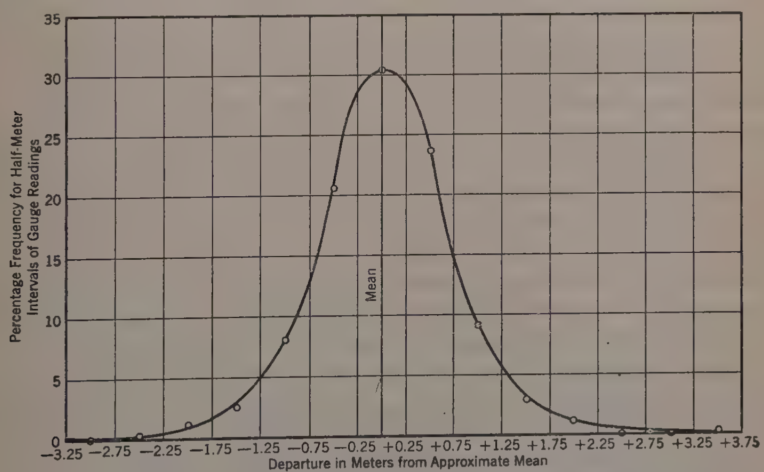

FIG. 4.—FREQUENCY CURVE, MAXIMUM READINGS, RODA GAUGE, CAIRO, EGYPT.

In Fig. 4 the effect of progressive change and discontinuities has been eliminated to some extent. The ordinates give the percentage of cases in

[36] "Nile Control", by Sir M. MacDonald, Cairo, Govt. Press, 1921.

which the reading falls within the half-meter indicated by the abscissa and, therefore, an ordinate is a measure of the probability of occurrence. The plotted points are derived from observations from 640 to 1451 A. D., together with observations from 1737 to 1917 A. D., making a total of 961 observations. It will be seen that the frequency distribution of Nile floods approximates closely that given by the Gaussian law of errors, and the regularity of the curve tends to show that systematic misrepresentation of the records of low floods did not occur to any great extent, and that perturbing effects act mainly in an accidental manner.

In the paragraph following Table 1 of the paper there seems to be a misconception of the effect of the Delta Barrage below Cairo. The function of this structure is purely to raise the river level so that the three main canals taking off at the Barrage can be fed at all times of the year. The storage behind the Barrage is negligible and, therefore, so is its effect in lowering the flood-peak or augmenting the minimum flow. In referring to storage and release of water in large reservoirs farther up stream, however, the author's conclusions are correct.

In recent years considerable archæological research has been done with regard to Lake Moeris and the Fayum Basin and the results have been published.[37] A few references to old records of the Nile are: (1) "Nile Gauge at Roda", by Mohammed Kasim[38]; (2) "The Nilometer at Cairo", by Dr. Reiss[39]; and (3) sundry items of correspondence on nilometers.[40]

Thomas H. Means,[41] M. Am. Soc. C. E. (by letter)[41a]—Attention is drawn, in this paper, to the long record of the Nile. Many references have been made to such records and there have been a number of summaries and brief discussions of the data, but the records themselves have not been available to many engineers in America. Mr. Jarvis has presented in pictorial form about as much as most engineers will want to see, while the specialist can go back to the more detailed data. The author has done another "good turn" in his studies of the reconciliation of several sources of data, thereby supplying some of the omissions in the Roda record, and in his explanation of the conditions under which the records were collected and of the probable reasons for some of the errors that may be included.

These records are of great importance because, aside from tree rings, they are the longest known continuous record of any kind of information that bears upon the changes in river flow or variations in climate. Briefly, they show the yearly high and low level of the Nile for 1 300 yr—information of great interest to hydraulic engineers the world over. The gauge from which

[37] "The Desert Fayum", by G. Caton Thompson and E. W. Gardner, The Royal Anthropological Inst., 1934; see, also, the review of this book in the *Geographical Journal*, August, 1935.

[38] "Nile Gauge at Roda", by Mohammed Kasim, Cairo, Govt. Press.

[39] "The Nilometer at Cairo", by Dr. Reiss, *Zeitschrift für Messungowesen*, 1889, pp. 439–445 (Konrad Wittwer, Stuttgart).

[40] Correspondence on Nilometers, *Proceedings*, Royal Geographical Soc., Vol. XI, No. 4, April, 1889, pp. 245 and 246.

[41] Cons. Engr., San Francisco, Calif.

[41a] Received by the Secretary November 9, 1935.

these records were obtained is located on the Island of Roda, at Cairo, and no doubt has been inspected by many engineers who have been fortunate enough to visit Egypt. The substantial character of the structure and of the many repairs and improvements that have been made upon it, and the care with which records have been kept through the more than thirteen centuries, speak eloquently of the importance of the level of the Nile to the people of Egypt. No nation has been so dependent upon one source of water and in no other place in the world is there so much interesting material for profitable study by the engineer.

The recent series of dry and hot summers in many parts of the United States has attracted more than usual attention to matters of weather. In parts of the Pacific Coast there have been seventeen years with precipitation below the average—below the normal. The question is, "Do we know what is normal?" "Is this dry period normal or were the earlier years of relatively high precipitation more nearly normal?" News items have told of railroads hauling drinking water for stock in Imperial Valley. Cities and towns in the Mississippi Valley have placed restrictions on the use of water, and thousands of fertile acres are suffering from lack of rain. An "unprecedented drought" is the explanation. Such disasters occurred not only in 1934, but for several preceding years. Are they "unprecedented"? Are they likely to occur soon again? The climate is changing, say some.

The belief that the climate is changing has been adopted as probable by many people and sensational broadcasts have told of the necessity of abandoning large areas of the great plains. The people are told that they have overdeveloped their water resources and must now take a backward step, consolidating their dwindling resources to fit the true conception of the water supply or rainfall.

The memory of human beings is notoriously unreliable as to weather. The deep snows of the past generation, the freedom from floods, or greater floods, the "big freeze", or the "big wind" are phenomena which, to human recollection, no longer occur. Examinations of records of weather, when they are available, seldom support these beliefs. Actual records seldom show any progressive change within historic times. Long-term records, such as these from the Nile, enable the student to carry facts a little further back, to compare with beliefs and theories.

When one reads of abandoned cities in now desert places, or sees wave-cut shore lines made by prehistoric lakes now long evaporated and either gone or greatly reduced in size; when one sees in the abandoned trails of glaciers proof of changed climate, one wonders, if such changes are going on to-day.

There are no weather records in America of any high degree of accuracy extending over 200 yr; in Europe, of not more than 400 yr, which is a very short time when compared with the 4 000 or 5 000 yr in which Man has gathered some historical facts, and insignificantly small when compared with the length of the most recent geological period. Estimating the length of geological time in years is by no means accurate, but from all the evidence available it appears that the time since the last glacial period is from 25 000 to 100 000 yr. The longest rainfall record, therefore, is in the order of

one-twenty-fifth of the historical period, or one-five-hundredths of the most recent geologic period. If the rainfall in the eastern part of the United States was reduced one-half in quantity since the last glacial epoch the rate of reduction would be about 0.1 in. per century. There is no way of ascertaining with certainty any such small average change in rainfall. So far as local records alone show, this country may be in an increasing or decreasing cycle—no man knows which. The records of the Nile, therefore, are of great interest in extending the period of observation. Perhaps if it were 4 000 or 5 000 yr long, the trend could be predicted.

The height of the Nile at flood time, in a general way, is dependent upon rains in the Abyssinian highlands; the low flow depends on rains in Central Africa which reach the main stream through the White Nile. There are many factors other than rain that may effect both these flows but, on the whole, rain is the most important single influence, and in a long period, such as this, its effects must have over-shadowed all other factors.

The Nile is an unusual stream. It flows out of a tropical region including in its water-shed humid, semi-arid, and desert areas of great size. The total area of water-shed approximates 1 200 000 sq miles, or nearly one-half the area of the United States. Its yearly flow in volume approximates 75 000 000 acre-ft, or about as much as all streams in California combined, or one-half of the Columbia River at The Dalles, Ore. The flow in Egypt, ordinarily, is at a minimum in May when it begins to rise from rains in Ethiopia; first the Atbara, then the Blue Nile rise in flood. The flood increases, reaching a maximum in September or October to fall as the Ethiopian summer rains diminish. The White Nile maintains the flow during the winter until the summer rains come again. In round numbers, two-thirds of the flow comes from the Blue Nile, one-fourth from the White Nile, and one-eighth from the Atbara.

In this manner the Nile has been rising and falling each year for several thousand years. In ancient times the cause of this regular procedure was a mystery satisfactorily explained only by introducing the supernatural. Then, nearly the entire irrigated area in Egypt was flooded in basins, under more or less control, as the river rose. In years of very high floods the control was ineffective, and disaster and famine followed; in years of low flood, the supply became inadequate and equally great disaster resulted. The reports accompanying the records published in the *Memoirs* of the Institute of Egypt give the harrowing story of these periods. On the whole, however, the rise and fall of the stream were so regular and dependable that Egypt became one of the richest countries in the world and was the storehouse of food from which neighboring countries were supplied in time of drought and famine.

It was the custom of the rulers to announce each year the time when the river had reached the proper level for the dikes to be opened and the basins flooded. This "wafa" was the occasion of rejoicing and feasting, the important period of the year, for on it depended the prosperity of the country and the amount of taxes which could be collected.

The records of flood level—presumably the highest level on the Roda gauge—and the low-water stage at the same place then became one of the

most important records in Egypt and probably epitomizes the economic history of the country. It is no wonder such care was taken in their preservation.

The Roda gauge was in its present location as early as the Seventh Century although there is reason to believe that earlier gauges existed either here, or at Memphis, a few miles up stream. The gauge was built in a temple, or mosque, and the collection and preservation of records was the duty of the priest or sheikh of the temple. The record now existing begins in 622 A. D.

A summary of the thirteen centuries of record is given in Fig. 5. This includes only the Roda gauge readings and not the additional data recorded

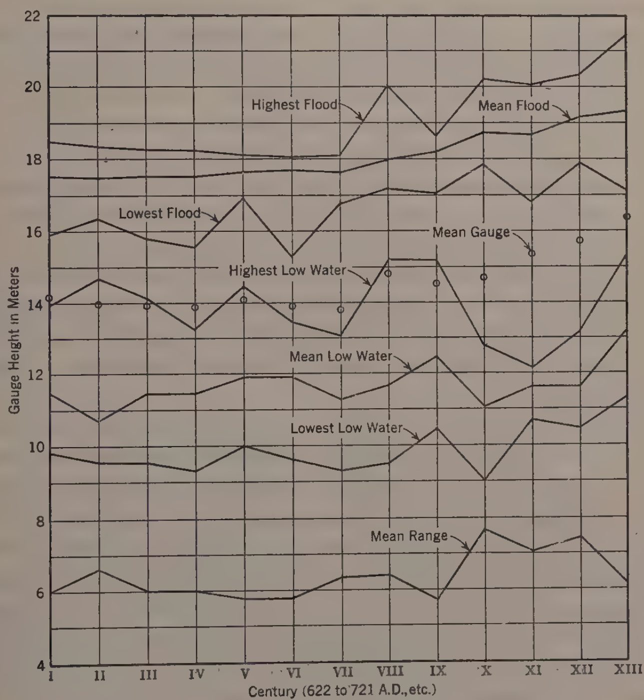

FIG. 5.—RIVER NILE AT RODA GAUGE, CAIRO, EGYPT.

in Mr. Jarvis' charts. This record of two readings of river level—the highest and the lowest— in each year does not give information from which the yearly flow may be computed but, in a stream like the Nile which goes through a regular cycle, such figures are very significant and should indicate any progressive change that may be taking place.

Little information is available concerning changes in the size of the channel, but such as there is indicates that there has been no notable variation in the width of the waterway. From Aswan to the head of the Delta near Cairo the stream seems to have occupied much the same channel since the Egyptians began to build dikes and cities and temples. The location of prominent objects and many historical references all indicate no great change in channel width.

The river levels at Cairo might have been effected by changes in the irrigated area in Upper Egypt, but so far as is known from historical records there was no important change in either area or method of irrigation in the thirteen centuries, until 1833, when Mehemet Ali began the construction of barrages at the head of the two branches of the Nile a few miles below Cairo. This structure effected the levels at Roda by back-water. Prior to 1833, the stream flowed past Cairo substantially as it has done for many centuries. In ancient times Lake Moeris, one of the "seven wonders of the world", may have effected the levels. This lake, which occupied what is now known as the Fayum Basin, a depression in the Lybian plateau west of the Nile Valley, was filled at high water and drained as the flood receded, thus affording both flood protection and water supply to the valley below. It is believed that this lake was not used after 300 A. D. Certainly, it has not influenced the river during the period for which the Roda records are available.

The silt deposited by the Nile has been building up the level of the valley since the dawn of history. This effect is very noticeable throughout the length of the valley. The earliest scientific study of this, of which there is a record, was made by the savants who accompanied Napoleon to Egypt in 1798. They estimated from the depth to which structures were buried that 10 to 12 cm (4 to $4\frac{3}{4}$ in.) of soil were deposited per century. More exhaustive research, with other approximate information considered, has confirmed this figure and, to-day, it is generally accepted that the mean rise is about 13 cm, or 5 in. per century.

The gauge heights at Roda, for the thirteen centuries show nearly the same rise, both in high water and low water, disregarding the period since 1833. This fact is very significant and indicates that there has been only one change in conditions and that corresponds to the rise from silt deposition.

In the ages in which historical records have been preserved the number and location of the arms of the Nile has been changed several times, and there has been a constant building out of the Delta, extending the channel length. The swamps which once made up the Delta, have been built up by silt, dikes have been constructed to control overflow, and the entire character of the country has been changed; all of which is in agreement with what one would expect from a silt-bearing stream.

The entire Nile Valley, from the first Cataract at Aswan to the Mediterranean, is formed of sediment. The river encounters no rock after leaving the granite at Aswan. The slope of the water surface as far as Cairo is remarkably uniform both at low flow and in flood; it is about 1 on 13 000, or a little less than 7 in. per mile. The cross-section of the stream bed is also very uniform, increasing slightly down stream. On the whole, the result is what might be expected from a silt-bearing stream flowing through a channel of its own deposition. As the valley level rose through deposition, the stream has maintained its channel in depth, width, and slope.

Since the change in average levels on the Roda gauge corresponds so closely to the general average change in valley levels there is no reason to believe that there has been any progressive change in volume of flow.

These records from the Roda gauge indicate the following conclusions:

(1) There has been a fairly constant rise in levels, which may be attributed to the sediment building up the valley lands and the river bed at the same rate. Disregarding the last century of records (which since 1833 has been effected to some extent by work on the barrage at the head of the Delta), there has been an average in flood levels of about 14 cm, or $5\frac{1}{2}$ in. per century, in the 1 200 yr.

(2) The mean flood, taken as the average of high flood and low flood, has increased at about the same rate.

(3) The mean height of low water has apparently increased at a smaller rate, although the greater variation in low flow renders determination of the change uncertain. This may have been caused by the regulating effect on low flow by diversions, or by the dams from vegetation in the Sudd or swamp region on the White Nile.

(4) The mean of all records, averaging mean flood and mean low water, has changed about as the mean flood level, an increase of about $5\frac{1}{2}$ in. per century. The change in the first seven centuries was less than in the latter five centuries.

(5) The mean range showed little change for nine centuries; then it increased about a meter for three centuries, but on the whole, it changed little in the 1 300 yr. This seems to indicate little change in the rise and fall of the river as a whole and points to a comparatively uniform channel during that period.

(6) There is nothing in this 1 300-yr record that shows any progressive change which may be attributable to change in climate.

J. W. BEARDSLEY,[42] M. AM. SOC. C. E. (by letter)[42a].—Based on extensive records, this paper is complicated by the need of changing units and the interpretation of various languages. The Italian, Lombardini, stated in 1865 that "no river in the world lends itself to hydrological studies on so majestic a scale as the Nile." The Nile is Egypt. British engineers have estimated that it built up the Delta at the rate of about 6 cm per century (see Fig. 6 (*a*)). It furnishes ample water for fertile lands in a region practically rainless. In 1908, Sir William Willcocks, referring to the old basin irrigation, wrote:

"It will be an evil day for Egypt if she forgets that, though basin irrigation with its harvest of corn has given way to perennial irrigation with its cotton fields, the lessons which basin irrigation has taught for 7 000 years can be unlearned with impunity. The rich muddy water of the Nile flood has been the mainstay of Egypt for many generations, and it can no more be dispensed with today than it could be in the past. * * * The basin irrigation of the ancient Egyptians may well be likened to the path of the eagle in its boldest flight, while the perennial irrigation of our day finds its true simile in the laborious task of the working bee."

Exclusive of Fayum and a few square miles of irrigable patches along the Nile between Aswan and Cairo, the productive area of Egypt is an equilateral

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[42] Cons. Engr., Syracuse, N. Y.

[42a] Received by the Secretary November 16, 1935.

triangle with sides about 240 km (see Fig. 6 (*b*)) long and with its apex at Cairo, approximately one-fifth of the area of New York State.

The White Nile rises in Lake Victoria under the equator, about 5 600 km from the sea, flows through long stretches of the Sudd with its papyrus, its swamps, and marshes, and delivers clear water into the Nile at its junction

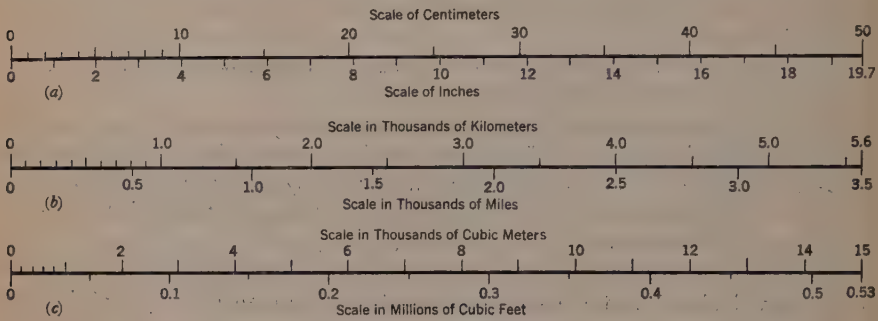

FIG. 6.—CONVERSION SCALES.

with the Blue Nile at Khartoum, 3 036 km from the sea. The Atbara joins the Nile about 320 km below Khartoum, and these two tributaries are the only ones between Khartoum and the Mediterranean Sea—a solitary course of 2 768 km to the sea.

The total drainage area of the Nile is about 2 867 600 sq km (501 000 sq miles), of which the White Nile drains about 60%, the Blue Nile and the Atbara about 10% each, and these last two rivers bring down the floods of muddy waters during August and September; otherwise, their discharge is nominal. A table of monthly discharges prepared by Mr. E. M. Dawson[43] covers the years 1890 to, and including, 1908, the discharges being taken at Sarras 390 km south of Aswan. The mean low flow for May, the lowest month, was 800 cu m (see Fig. 6 (*c*)) per sec, ranging from 540 cu m in 1907 to 1 410 cu m in 1894. The high stage of September, the highest month, averaged 10 470 cu m, varying from 6 870 cu m in 1900 to 14 590 cu m in 1893.

An inspection of Fig. 1, covering maximum and minimum stages for 1 310 yr, suggests that equatorial cycles of rainfall may be shown by a study of minimum stages, the clear waters of which are derived almost entirely from the White Nile. A casual examination indicates cycles between 30 and 40 yr, or multiples thereof. Such an opinion needs verification based on original records. An expression of opinion by Mr. Jarvis regarding such verification would be of considerable interest.

The writer's notes are based on investigations made during the spring of 1909, when the height of the Aswan Dam was increased by 5 m. At that time a photograph was taken of the nilometer on Roda Island, near Cairo (see Fig. 7). This old gauge of Nile floods, a granite-like column with an ornamental capital, is located in a well about 16 ft square. It is said that when waters flooded the capital, benefits were a maximum and so were taxes. When flood waters did not reach the ornamental capital, or when they cov-

[43] *Cairo Scientific Journal*, October, 1908.

ered it entirely, taxes were reduced proportionately. Insufficient water reduced the normal crops. The excessive flood waters were harmful and taxes were reduced accordingly. The diameter of the gauge column is now (1935) estimated to be from 18 to 20 in. This gloomy well in a neglected garden, covered with a crude roof of galvanized iron supported by four posts, with its narrow steps growing slimy opposite the beautifully carved capital of the gauge column, and with a black bottom of unknown depth, was not specially inviting, but the view obtained was well worth the trouble. The date of the construction of this gauge was not determined. It was said that when floods reached the capital, agricultural conditions were perfect. Higher floods caused damage and lower floods were deficient. Land taxes were levied proportional to the stages above or below the capital, and if such stages were excessive, no taxes were levied. A maximum tax was due when the flood-stage was on the capital, a range of about 0.75 m. The low stages of May, and the rapidly increasing floods during July and August, and the September peak[44], are clearly shown on Fig. 8.

Fig. 7.—The Nilometer on Roda Island, Near Cairo, Egypt.

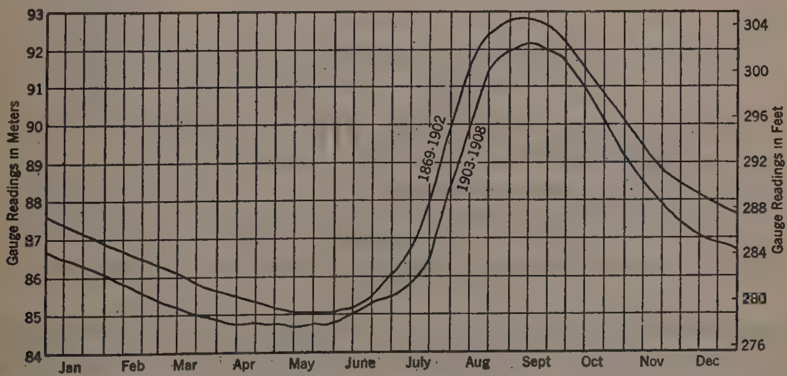

Fig. 8.—Gauge Readings at Aswan Dam, 1869–1909.

Possibly, the Yellow River of China has as many features in common with the Nile as any other river in the world. It flows through a fertile delta built

[44] Pub. by the Survey Dept., Cairo, Egypt, March, 1909.

up during past centuries by its load of silt. It flows through a region of scant precipitation insufficient for sprouting spring plantings, and the dense population is dependent upon irrigation. Its tributaries are small and few throughout its 900-mile course through its deltaic area from its western mountain barriers to the sea. It also has long-time records of floods, which are complicated, however, by the occasional destruction of controlling dikes. Its seasonal floods lack the regularity in time and volume of the Nile floods. In contrast with the Nile, which flows north and is entirely free from ice, the Yellow River flows easterly and through some regions of intensely cold winters.

A study of the Yellow River might develop hydrological facts of value somewhat similar to the author's study of the Nile.

J. C. Stevens,[45] M. Am. Soc. C. E. (by letter)[45a].—These records constitute some of the most interesting long-time river data that have been published, and the author certainly deserves the thanks of the profession for presenting them.

The writer is particularly interested in the rate at which the Lower Valley of the Nile has been raised by sedimentation as disclosed by these records. He is also curious to know whether any cyclic variations are in evidence.

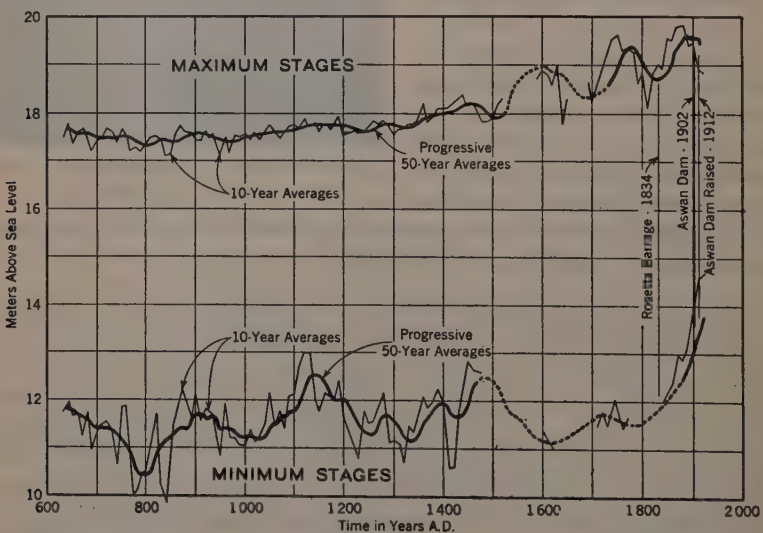

Fig. 9.—Maximum and Minimum Heights of River Nile at Cairo, Egypt, 622 to 1933 A. D.

Upon request, Mr. Jarvis kindly supplied large-scale prints of the data in Fig. 1. From these the average gauge height for each decade was read and tabulated. Progressive 5-decade averages were then computed. The results are shown in Fig. 9.

[45] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[45a] Received by the Secretary November 25, 1935.

The light lines are 10-yr averages read directly from the prints. The heavy lines are the progressive 5-decade averages—that is, each point on the heavy line is an average of the preceding five decades. The upward slope of the heavy lines may be taken to be an index of the rate at which the Lower Valley of the Nile is being raised by sedimentation. Neglecting the increased stages caused by the construction of the Rosetta Barrage and Aswan Dam the average rate of rise is practically 4 in. per century for both the maximum and minimum stages.

The highs and lows are prominent in the minimum stages and are nicely demonstrated by the progressive 5-decade line. No periodic cycle is in evidence, but high cycles alternate with low cycles of irregular duration.

Another significant point is that as evidenced by this nearly continuous record and from the sporadic records of isolated periods running back over 5 000 yr, there appears to have been little or no climatic changes that can be detected. In other words, it appears that the regimen of the Nile at present is very much as it was about 5 000 yr ago.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DISTRIBUTION OF STRESSES UNDER A FOUNDATION

Discussion

BY MESSRS. JACOB FELD, GEORGE PAASWELL, AND G. S. SALTER

JACOB FELD,[34] M. AM. SOC. C. E. (by letter).[34a]—The subject touched by this paper really involves two major classes of problems in foundation engineering, namely:

(1) For a foundation unit of a given size, both in area and in shape, carrying a known load, either as a point concentration or as a distribution of known variation: (*a*) What is the distribution of stress at the base of the foundation unit or footing? (*b*) what is the variation in the distribution with changes in the assumed relative and also absolute (for end conditions or cases) stiffness of the footing and of the supporting soil? and (*c*) what is the variation in the distribution with increases in the loading, for a given ratio of stiffness of footing and soil, including the end condition where the loading causes a change of the soil, or of the footing, from elastic to plastic material?

(2) For a given load on a footing of given size, the distribution of stress at the base having been determined: (*a*) What is the distribution of pressures through the soil? and, (*b*) what changes in distribution occur with increased loadings?

In all the foregoing problems, it must be remembered that the summation of stresses from various loads can only be made as long as the total does not exceed the elastic limit. As soon as plastic flow (in clays), or failure in shear (lack of tension in sands), occurs, the problem changes.

The author has noted clearly that his paper covers only one case of the general problem, namely, what is the distribution of stresses along the vertical axis of the soil supporting a circular load for: (*a*) Uniform; and (*b*) parabolic distributions of stress at the base of the circular footing? He shows definitely

NOTE.—The paper by A. E. Cummings, Assoc. M. Am. Soc. C. E., was published in August, 1935 *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: October, 1935, by Messrs. Clement C. Williams, D. P. Krynine, and L. C. Wilcoxen; and November, 1935, by Messrs. Marshall G. Findley, and M. A. Biot.

[34] Cons. Engr., New York, N. Y.

[34a] Received by the Secretary November 14, 1935.

that the assumption of parabolic distribution gives theoretical results which agree with the experimental work with sand, in which all the conditions of the problem are met.

As an opening wedge, the paper is valuable. It is hoped that the author's warning (see "Summary and Conclusions") will be heeded: "These formulas can be used only when the practical problem fulfills all the conditions under which they are derived."

The theoretical formulas are based on the assumption of action in isotropic bodies. The definition of the term, "isotropic", is "having the same physical properties in all directions." This condition eliminates all soils except for very small loadings. In general, the approximate maximum load that can be considered, is the largest load which will not cause a permanent set or deflection after it is removed.

All the tests enumerated in Table 1 are for sand and circular disks. No attempt has been made to correlate the effect of the amount of unit loading with the variation or law of distribution in the soil.

In the tests by Messrs. R. B. Fehr and C. R. Thomas[35] on square and rectangular test areas, for sand, clay, and loam, a definite change in distribution was noted with increased loadings.

The very high unit stresses found in the area at the center of, and just below, the circular disk footing is not serious because in the actual condition there will be a release of support and, therefore, a redistribution of loading in this area. The theoretical solution of the problem is equivalent to a first approximation of a statically indeterminate structure in which deflections are neglected. In addition, the center core of soil under a footing is better retained against lateral movement and, consequently, against vertical movement than the remainder of the soil body. The effect is to permit greater loading without large settlement or flow.

Where long loadings, such as quay walls, on soft ground, give high stress concentrations on a long strip of the supporting soil, with consequent settlement and flow of the mud or silt and often failure of the wall, it has been found most economical to eliminate the soft soil directly beneath the wall and replace it with a layer of sand. The entire picture of stress distribution in the soil is then changed not only because the high localized stress at the base of the wall is taken by the sand, but also by the formation of a new soil at the contact zone. A very fine description of the methods and large scale tests on this method of solving the problem is given by Gen. Carlo Barberis.[36]

Under "Numerical Example", the author points out that although greater areas of soil are affected at increasing depths below a loaded footing, the distribution at any depth is not uniform. The assumption of parabolic distribution, made in this example, may apply to sand, but certainly should be modified if clay or any cohesive material is used, chiefly because of the edge

[35] "Experiments on the Distribution of Vertical Pressure in Sand", by R. B. Fehr and C. R. Thomas, *Bulletin No. 8*, Pennsylvania State Coll., 1913; and "Further Experiment on the Distribution of Vertical Pressure", by R. B. Fehr, *Bulletin No. 10*, Pennsylvania State Coll., 1913.

[36] "Recent Examples of Foundations of Works, Such as Quay-Walls", etc., by Carlo Barberis, XVI International Congress of Navigation, Second Section, Brussels, 1935, "Ocean Navigation", Third Communication by Carlo Barberis, Rome, Italy.

effect on the perimeter of the footing. For all soils, the assumed distribution will change as soon as a deflection occurs.

The true distribution at the base of a footing is such that the sum of the works performed in bending, in shear, and in compression of the footing is a minimum, subject always to the condition that no stress exceeds the elastic limit. It would be a very unusual design problem which would warrant such a computation. Probably, photo-elastic methods to show the qualitative conditions would be advisable to reduce the number of possible distributions that should be studied.

GEORGE PAASWELL,[37] M. AM. SOC. C. E. (by letter).[37a]—It is gratifying to note the interest in the development of accurate methods of obtaining pressure distributions under foundations. The mechanics of foundations is predicated upon a correct determination of stress distribution in the underlying strata. However, correctness may be defined herein not as a rigorous mathematical solution consistent with all the data, but rather as a general correctness of stress distribution over the broad extent of the strata, subject to distortion by pressure of the foundation loading. In this sense of the word, the "local pertubation effect" immediately adjacent to the point of loading is of no significance in the general stress distribution. The problem may be viewed as analogous to the stress determination in a girder subject to concentrated loads, where the effect on the point of application of the load on the local part of the girder is considered of secondary importance, the general distribution of moment and stresses being the primary problem involved. The manner of loading a foundation and the soil pressures near its loading can generally be ignored in the determination of the stress distribution in the deep strata the distortions of which will usually determine the final movement of the supported structure. The analysis as presented by the author—although of interest to the general theory of stress distribution—should not leave a serious doubt in the reader's mind as to the general validity of the Boussinesq equations. These equations, as given in the report[6] of the Society's Special Committee on Earths and Foundations, are sufficiently rigorous and consistent with experiment to permit a scientific prediction of foundation behavior. Their use in foundation design involves the same degree of accuracy as the use of the ordinary Bernoulli formulas for flexure.

It is unfortunate that the development of the Boussinesq equations is buried in the intricate treatises on the mathematical theory of elasticity. The fundamental assumptions of the theory of elasticity are that: (1) All displacements are infinitesimal; and (2), Hooke's law is correct.

Boussinesq was fortunate in discovering an ingenious device for the solution of the differential equations for displacement of elastic solids under a surface load. A function is found, such that its derivatives express the displacement of a given point under a certain type of surface loading. By

[37] Red Wing, Minn.

[37a] Received by the Secretary, November 25, 1935.

[6] Progress Rept. of the Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May 1933, p. 780.

combining two forms so as to induce the surface loading to a single force at a point, the displacements are evaluated, and from these the forces are found. The force system in horizontal planes (if the applied load is vertical and the boundary plane horizontal) involves no elastic constants. It so happens that the force system on vertical planes contains elastic constants, which, for the usual soils, permits values that cause the terms containing these constants to vanish, so that it may be stated, in general, that the force systems on both vertical and horizontal planes do not involve any elastic constants so far as foundation problems are concerned.

G. S. SALTER,[38] ASSOC. M. AM. SOC. C. E. (by letter).[38a]—The problem of the distribution of stresses in a foundation is quite complicated, and much good work has been done on the subject in the last few years, to which this paper is a commendable addition.

The writer has found that most of the investigations on this subject, both theoretical and experimental, have dealt with uniform or parabolic load distributions on the footings (either actual or assumed), whereas in so much of the foundation design encountered in his work the loading is line distribution on semi-flexible footings; also, while most of the literature deals with the determination of stresses at some distance below the contact surface, in many cases (especially on a clay varying from medium stiff through the various gradations to hardpan), the problem is more to determine the pressure distribution directly under the footing than the pressure intensities at some lower depth where the clay is more compact.

It has long been realized that variations in conditions lead to entirely different pressure distributions and that those resulting from line distribution on a semi-flexible footing, such as a heavy wall on a relatively thin slab, give high pressures directly under the wall.

The writer has made some investigations in this field and has come to the conclusion that the following procedure is a logical method of dealing with this particular problem. Make soil-bearing tests to determine settlement for different loading values and then determine, by trial if necessary, such pressure distribution values on the contact area between the footing and the soil so that the differential settlement resulting from the varying soil pressure equals the flexure in the footing.

Such line loads give a contact loading distribution somewhat parabolic in character, the variation depending upon the load-carrying capacity of the soil and the relative weights of the wall and the slab. Sometimes, it has been desirable to make various assumptions (within reason) on the bearing capacity of the soil and to design for the worst probable condition. Pressure distributions thus found are far from the uniform pressures commonly assumed, and where footings are somewhat flexible, or when the soil has high load-carrying values, a line load may be distributed over only a small portion of the footing directly under the load.

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[38] Structural Designer, The San. Dist. of Chicago, Chicago, Ill.

[38a] Received by the Secretary November 29, 1935.

The foregoing is only one of the many problems on the behavior of foundations and is presented in an endeavor to show, as the author has stated, that, as a general rule, it is not satisfactory to assume that the pressure at the surface is uniformly distributed.

For the general problem of stress distribution which exists in the ground under a foundation to which the theory of Boussinesq applies, the University of Illinois has recently published a circular on simplified computations of vertical pressures.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

THE RELATION OF ANALYSIS TO STRUCTURAL DESIGN

Discussion

By Messrs. L. J. Mensch, and Russell C. Brinker

L. J. Mensch,[7] M. Am. Soc. C. E. (by letter).[7a]—The gradual increase in scientific observations of engineering structures has led to an increasing attention to details of theory. The normal action, or what may be more correctly termed the idealized action, of structures as described and taught by engineering textbooks occurs rarely, and very often other influences cause deformation and participation stresses that becloud the issues in actual practice.

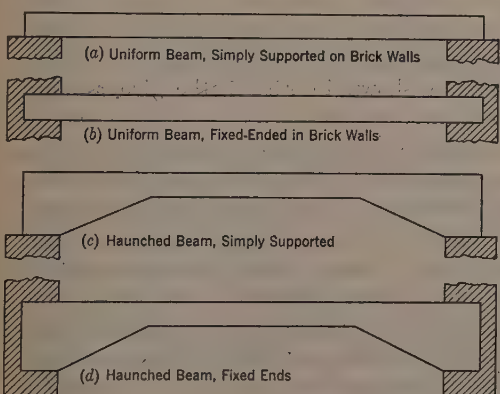

Fig. 1.

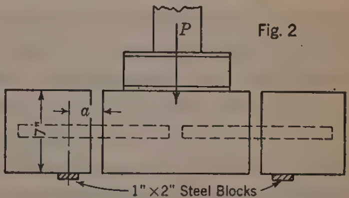

Fig. 2

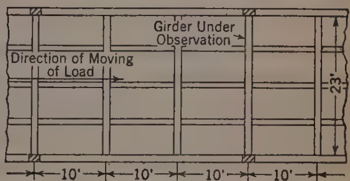

Fig. 3.

A good illustration of this point is afforded by tests made by Dr. F. von Emperger on a large number of girders for the Austrian Concrete Committee.[8] In Fig. 1, (*a*) represents girders loosely supported on brick walls; (*b*) represents girders well embedded in brick walls; (*c*) represents girders with

Note.—The paper by Hardy Cross, M. Am. Soc. C. E., was published in October, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[7] Civ. Engr. and Constructor, Chicago, Ill.

[7a] Received by the Secretary November 7, 1935.

[8] "Mitteilungen über Versuche", Heft IV, 1913.

brackets loosely supported on brick walls; and (*d*) represents girders with brackets well embedded in brick walls. All girders had the same span of about 13 ft, all had the same cross-section of $6\frac{1}{2}$ by $8\frac{1}{2}$ in., and the same reinforcement, except that some beams had brackets at the ends, as shown. Girders (*c*) failed at 75% more load than Girders (*a*), although according to the normal action as defined by textbooks the ultimate strength should have been the same. Girders (*b*) failed at 250% more load than Girders (*a*); and Girders (*d*) failed at 575% more load than Girders (*a*). The participation stresses as classified by Professor Cross were of a favorable nature in this case.

A similar effect was observed[9] by the writer in 1934 when having tests made on dowels connecting concrete blocks, as shown in Fig. 2. The stresses in the dowels at ultimate load, when computed by the formula, $s = \frac{P\,a}{2}$, were from 150 000 to more than 200 000 lb per sq in. As the re-rolled steel was probably of less strength than 100 000 lb per sq in., the effect of the friction at the supports was to increase the strength of the dowels 100 per cent. Both favorable and unfavorable participation and deformation stresses are found in concrete pavements, due to friction on the subgrade. Friction causes the concrete to crack when the pavement shortens, due to temperature and moisture variation, but when a concentrated load deforms the under side of the slab, the friction causes a moment of opposite direction to the moment of the concentrated load.

A very thorough investigation of a similar nature may be found in the report of the French Committee on Reinforced Concrete[10] The Committee tested to destruction, a reinforced concrete floor of the Science and Art Building of the Paris Exhibition of 1900. As shown in Fig. 3, the girders were of 23-ft span, placed 10 ft on centers; they were connected by small beams of 6 by 8-in. stems, and covered by a 4-in. slab. Concentrated loads of from 10 000 to 42 000 lb were pulled on a small truck in the center line of the building, and the deformation and deflection of one girder were observed when the concentrated load was at various distances from the longitudinal axis of the girder under observation. At a distance of the concentrated load from the girder of 30 ft, 20 ft, and 10 ft, the deflection of the girder was 1%, 10%, and 47% of the deflection observed when the concentrated load was exactly over the girder, and the Committee concluded from its many observations that the stresses produced in the last case were 50% of those of a single girder, disconnected from the adjoining girders.

One can readily infer from these examples that secondary stresses often help the structures to carry higher loads, and so-called inventors have often misled the public by taking advantage of this fact.

The writer cannot agree with Professor Cross that the post and lintel structure is a statically determinate system. The lintel will bend under load and throw the upper inside edges of the posts into heavy compression, thus producing an eccentricity of the load on the posts for nearly one-half

[9] "Joints for Concrete Pavements", by L. J. Mensch, 1935, p. 33.
[10] "Experiences, Rapports", etc., 1907.

their depth. As a consequence, the columns may and may not bend sufficiently to produce at the top an inclination equal to the rotation of the lintel at the ends and thereby offer an even bearing to the lintel. The analysis of such a system has been given by the writer elsewhere[11].

All properly designed bents with either slender or stiff columns may be classified as good engineering if they can be defended on architectural or economical grounds. One may consider all deformation and participation stresses, the deflections of the girders and columns, the flow of concrete due to dead weight, the flow of concrete due to the interaction of concrete and steel during shrinkage, temperature effects, active and passive earth pressure, and possible movements of the abutments. Nevertheless, to make a structure lasting and free from serious blemish, there is still lacking a very important point to the beginner, and this is practical experience and critical observation of existing, similar structures.

In ordinary reinforced building construction engineers are accustomed to design the highly indeterminate girders and slabs by the bending moment formulas: $M = \frac{w L^2}{8}$; $M = \frac{w L^2}{10}$; $M = \frac{w L^2}{12}$; and $M = \frac{w L^2}{16}$, as given by the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete[12]. This Joint Committee did not give any rules for determining the bending moment on inside columns, and, therefore, many engineers believe that it is good practice to omit them from consideration. Exponents of scientific analysis of indeterminate structures may object to such formulas, violating as they do every rule of continuity and compatibility. They may object, furthermore, on the ground that such formulas do not seem to be based on logical reasoning; if they were logical, the coefficients would be different for dead load and for live load and would depend on the relative stiffness of the adjoining members. It is a fact, however, that only in rare cases (especially when there are very large differences of spans in the adjoining members) will a so-called exact analysis make a great change in the moments at the supports and the moments in the center of the girders will be found considerably smaller than those given by the Joint Committee. No great harm is done when the girders are made stronger than necessary in the central section and the adoption of these formulas has saved hundreds of millions of dollars to building owners. The introduction of this type of formula is to the credit of F. Hennebique, and dates from 1894. Structural steel engineers did not have such a leader and they are not in the habit of reducing the bending moment in girders of skeleton buildings from the moment of a freely supported girder.

Unless similar simple formulas are offered to engineers for the design of statically indeterminate structures such as those with two or more legged bents, rigid frame, and continuous bridges, old and new types of arched bridges, and single and multiple-arched bents for roof constructions, the writer is convinced that no truly great progress will be made in this field

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[11] *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1682.

[12] *Proceedings*, Am. Soc. C. E., October, 1924, Papers and Discussions, p. 1153.

in the United States. The writer has seen designs of that nature which cost considerably more than statically determinate structures. Collections of such formulas have appeared in handbooks in Europe and represent the labors of many conscientious workers, but they need much elaboration before American engineers will have sufficient confidence to use them often. The fact that only uniform sections are used for the various members works against their use for long spans; deflections are not given; proper details and examples are presented only sparingly and theoretical spans instead of clear spans are used in order to facilitate the computations. The work of A. Strassner, published more than twenty years ago, marks a great progress as it shows how to deal with members of varying sections. It marked the first attempt at improvement; it is still very theoretical, and the assumptions and the tables still contain errors. These tables have been used by American engineers for the construction of tables of other coefficients; the latter may contain additional errors and caution is necessary in using them.

Such a handbook as the writer has in mind cannot be written by a single expert or a college professor and his most capable assistants, but must be the work of a great many experts and their associates, and each problem must be investigated by at least four men, working, independently, in groups of two. There is no question but that the solution of statically indeterminate structures requires much tedious work. The most skillful mathematician is likely to make many errors, but two men working together can save time and avoid most errors. At present, only two methods are known for the solution of statically indeterminate structures: The mathematical theory of elasticity and the mechanical solution by apparatus such as those of Professor Beggs, Gottschalk, Nubupest, and others. The mathematical theory of elasticity states neither more nor less than that: Every particle of the interior of a body must be in equilibrium; every particle of the boundary surface must be in equilibrium; stress must be compatible with strain in every point; and, of course, the entire body must be in equilibrium under the action of the loads and other forces such as reactions and surface tensions (frictions and shear from adjoining bodies). It is practically impossible to solve the simplest case of a structural member according to the strict theory, and simplifications are necessary. A particle is in equilibrium when the sum of the components of all forces (stresses) acting on this particle parallel to the X, Y, and Z axes are zero. One simplification most often used is to disregard the stresses parallel to the Y and Z-axes and to confine the investigation to the stresses parallel to X-axis, which may be assumed to be in the longitudinal direction of the member. Another simplification which is often necessary is to investigate the stresses or conditions of equilibrium at only one or two sections of the member, and to hope and believe that nothing of a dangerous nature will happen in other sections or directions which are not investigated according to the rules of elasticity. A good example is Clapeyron's theory of continuous girders. Clapeyron established the conditions of equilibrium at sections over the supports by making the stresses parallel to the direction of the girder on

both sides of the section equal and of opposite direction. In Professor Cross' language he established the principle of continuity, but committed the inexcusable error of not considering the stresses due to the width of the support, which error changes the result by 20% in most cases.

In the traditional treatment of a fixed arch, the conditions of statical and elastic equilibrium are generally applied to only one section, and most easily to one abutment. The arch is considered as a cantilever fixed at the other abutment, and, under the assumed loading, the other abutment which is being investigated is assumed to deflect horizontally a distance, h, and vertically a distance, v; and it is assumed to rotate through an angle, b. The problem is find the horizontal force, H, the vertical force, V, and the moment, M_a that would cause the deflections, h and v, and the rotation, b, to vanish. In order to find H, V, and M_a, the cantilever is treated as a common girder and the horizontal and vertical deflections and the rotation of the free end abutment are computed for the loads and the unknown forces, H, V, and M_a. Then, the so-called elastic equations are formed by: Equating the sum of all horizontal deflections to zero; equating the sum of all vertical deflections to zero; and equating the sum of all rotations of the end to zero.

Not many careful tests have been made to establish how correct the designer is when he uses the formulas for the deflection and rotation of a straight girder of uniform section for the computation of the deflections and rotations of girders of curvilinear form with markedly varying section, knees-braces, or other irregularities. In extreme cases the error is probably of the order of 25 per cent. The identical equations can be obtained by the theory of least work, by the principle of virtual velocities, or by Maxwell's equations. The latter are simply rules for obtaining deflections and rotations; intrinsically they are not more scientific than the foregoing procedure and do not imply an "exact" theory of elasticity.

There are many methods of obtaining the deflections: By arithmetical summation, by integration, by the moment-area method, the slope-deflection method, and the shear-area method, to name only a few. These methods are not new theories of elasticity as some writers would like to have them called, but are more or less cumbersome methods of finding deflections. New methods are re-invented by every new generation of engineers in the desperate search for finding the nearly impossible—a simple method of computing deflections. Statically indeterminate structures would be used more often and more scientifically and would be much better understood if only one method had been taught in the last seventy years, and if that method had been thoroughly elaborated. The many methods described recently in literature are only a confusion to engineers.

In order to shorten the analysis of typical structures, engineers have computed the deflection and rotation of certain points on these structures for unit loads placed at various positions and have presented the results of their labors in the form of diagrams and tables, which are not yet simple enough to be of great use.

In the case of highly indeterminate systems with hybrid action, as termed by Professor Cross, such as trusses with double diagonals, skeleton girder and column constructions, and series of curved roof bents, the designer must deviate still further from the teachings of the theory of elasticity before he can find a way to analyze the structure. He must make new guesses or assumptions, and one of the oldest and best is that of Saint-Venant, who assumed that the local disturbance caused by a load at a particular point (or member) dies out at a short distance from the point of application. This guess, rule, or principle (some even call it a theory), when applied to a girder of a skeleton building, can be expressed as follows: Consider the girder to be analyzed as connected only to the adjoining spans and columns, the next adjoining spans and stories being considered as having no influence on the girder. The far ends of the adjoining girders and columns may be considered fixed or rotated at an angle suitable to the case. The surprisingly simple results were given by the writer many years ago.[11]

The mathematical difficulties imposed by the exact theory of elasticity (and very often by its simplified form) being insuperable, the analysis of most statically indeterminate structures is beyond the realm of science and becomes an art, in which experience, skill, imagination, guesses, and hypotheses, often of obscure origin, play a greater rôle than calculus, and where charlatans may run riot and false prophets may shine to the utter confusion of the student. A properly conceived handbook would elevate the art, which is still at a very low ebb.

RUSSELL C. BRINKER,[13] JUN. AM. SOC. C. E. (by letter).[13a]—A logical classification of the types of structural action and the stresses producing the different types is presented in this paper. Further emphasis might easily be placed upon the question of when secondary stresses may or may not be lightly neglected. The "dishing out" of floor-beam webs and the cracking of stringer-connection angles on some old railway bridges give evidence of secondary stress effects neglected in their designs.

The factor of safety, or more often the "factor of ignorance", allowed in the spread between the specification stresses and the elastic limit in steel bridge design to cover secondary stress and other effects could be put on a more rational basis if the secondary stresses were given greater consideration. Members having low percentages of secondary stresses compared to their primaries, are heavily penalized by giving them the same basic allowable unit stress as those members having high percentages. The fact that the secondary stresses have little effect on the ultimate strength of members does not mean that they should be allowed to produce greatly unbalanced total stresses below the elastic limit because this is the range of usefulness of most structural members. Truly more data on the question of when—or how—to discount these secondaries are needed.

Much refinement in analysis has been made in the decade, 1925–1935, but needed improvements in specifications covering unit stresses appear to have

[13] Instructor in Civ. Eng., Univ. of Hawaii, Honolulu, Hawaii.

[13a] Received by the Secretary November 14, 1935.

received little attention. It is obvious, that analysis is directly tied up with design stresses, as is the question of details (which is also somewhat behind in progress). In 1923 there was considerable discussion as to the validity of the unit stresses being proposed in specifications for railway bridges[14]. Many prominent engineers looked with disfavor upon the policy of using the same low unit stress for dead load as is used for live load to cover future increases, uncertain impact, etc., but little was accomplished. It is to be hoped that, delving into the types of stresses involved in structural action, Professor Cross will also revive interest in how to evaluate them in terms of more applicable design stresses.

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[14] *Transactions*, Am. Soc. C. E., Vol. 85 (1923), pp. 532 *et seq.*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PROPOSED IMPROVEMENT OF THE CAPE COD CANAL

Discussion

By C. S. Jarvis, M. Am. Soc. C. E.

C. S. Jarvis,[13] M. Am. Soc. C. E. (by letter).[13a]—On a project that required about 260 yr between its serious inception and the first construction work, and another 30 yr, or more, before it was opened to traffic at the outbreak of the World War, it is not surprising that many questions and problems require serious consideration before radical enlargements, revisions of alignment, and structural improvements of the Cape Cod Canal are authorized. The value of this project to navigation, as satisfactorily demonstrated during its first few years of operation, from the standpoint of time and distance saved as well as hazards avoided, led naturally to its being taken over by the Federal Government. To measure up to national standards for maintaining and promoting commerce by navigation, it was found necessary to enlarge the cross-section, revise the curvature and some other features of alignment, and to provide other measures of safety for vessels beset by fog and storm.

The restricted depth and width of the canal as originally constructed, together with the high velocities occasionally attained, due to tidal differences prevailing in Cape Cod and Buzzards Bays, were directly accountable for several mishaps to vessels undertaking to use the new waterway. Although the losses were relatively small as compared with the disasters avoided along the outside route, it became imperative to enlarge the cross-section and to improve the alignment of the canal. During the preparation of plans for this improvement, it developed that some vital differences of opinions existed among those responsible for the design, review, and supervision of the work; for example, it seemed to be regarded as axiomatic among some official circles, that the increase of cross-section would result in a considerable increase of velocity due to tidal action. To others, it was just as evident that there should be no pronounced change of velocity due to doubling or trebling the

Note.—The paper by Capt. E. C. Harwood, Corps of Engrs., U. S. A., was published in October, 1935, *Proceedings*. This discussion is printed in *Proceedings*, in order that the views expressed may be brought before all members for further discussion.

[13] Hydr. Engr., Soil Conservation Service, Washington, D. C.

[13a] Received by the Secretary November 1, 1935.

bottom width, with some increase in depth. In fact, the opinion was even ventured that if the widening were carried to great extremes, say 800 or 1 000 ft, the tides in Cape Cod Bay and Buzzards Bay would occur more nearly in synchronism, with decreased differences in water-surface elevation and velocities of flow. The apparent proof of this assumption is to be found in a consideration of what would be the condition if the two bays were connected by a very wide and unrestricted channel of great depth. Unquestionably, their tidal phases would thereafter be in fairly close agreement.

Investigations by Earl I. Brown, M. Am. Soc. C. E., and the late Harry Franklin Flynn, M. Am. Soc. C. E., in connection with the proposed New Jersey Section of the Intracoastal Waterway, to connect the Delaware and Raritan Rivers through a 31-mile land cut, demonstrated that enlarging the cross-section reduced the velocity of flow. The dimensions of the cross-sections compared were approximately those of the original and the recently authorized enlargement of the Cape Cod Canal, but somewhat greater in each case. If the aforementioned investigations and demonstrations are accepted as conclusive, it appears questionable whether the proposed enlargement to 540 by 32 ft will result in velocities already experienced in the original cross-section, or observed during the process of widening to the present 205-ft bottom width. From analogy, it seems to be fairly well assured that the wider section will accommodate a sufficiently greater volume of water interchanged from the two ends of the canal to reduce the tidal differences and the period of lag by measurable and considerable amounts, with a corresponding reduction in velocity of flow.

In view of the cost estimates published in Table 6, showing that the 540 by 32-ft canal section should cost approximately $4 300 000 less than a 250 by 32-ft canal with twin locks, it would be interesting to trace back among the earlier estimates in which the financial advantage was generally claimed for the lock canal. The project, the Engineering Profession, and the Federal Service are all greatly indebted to the Division Engineer of the North Atlantic Division, Col. G. M. Hoffman, Corps of Engineers, U. S. Army, who was largely responsible for the modification or reversal of prevailing opinion as to the proper procedure. His plea was for progress along lines that have been proved or that are readily susceptible of proof. As a result, the lock construction was delayed during the preliminary widening of the canal; and the intensive study of canal velocities as related to tidal differences in Cape Cod and Buzzards Bays resulted in significant data to prove that, with sufficient widening, the velocities would be found tolerable and the locks unnecessary. On the other hand, if the trend of evidence and opinion had continued to favor the construction of the locks after partial widening, then the structural work could have continued with more assurance, as the result of more mature consideration.

During the hydrologic observations of canal velocities simultaneously at designated stations, both before and after widening during recent years, notable progress was achieved in the art of gauging stream velocities with relation to water-surface elevations. In addition to the excellent controls introduced, as well described in the paper under appropriate headings, special

notice should be accorded the practical use and value of the automatic graphic recorder specially devised in the Engineer Office for such work. It was the writer's privilege, after participating in the preliminary studies, to observe the apparatus, some of the records, and the methods of developing and correlating the data. Unusual skill and ingenuity were displayed by those responsible for the operation.

The orderly presentation of historical and technical data relating to the Cape Cod Canal, especially at this time, is of great value to the profession. The author is to be commended for his painstaking effort.

PROCEEDINGS

R. T. Babcock

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American Society of Civil Engineers

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January 13-14, 1936:
A Quarterly Meeting will be held in New York, N. Y.

ANNUAL MEETING

NEW YORK, N. Y.

January 15, 16, 17, 18, 1936

January 15, 1936:
Morning.—Annual Meeting. Conferring of Honorary Membership, and Presentation of Medals and Prizes.
Afternoon.—General Society Meeting.
Evening.—President's and Honorary Members' Reception and Dance.

January 16, 1936:
Morning.—Technical Division Sessions.
Afternoon.—Technical Division Sessions.
Evening.—Entertainment and Smoker.

January 17, 1936:
All-Day Excursion to Rockefeller Center.

January 18, 1936:
Inspection Trips.

The Reading Room of the Society is open from 9:00 A.M. to 5:00 P.M. every day, except Saturdays when it is closed at 12:00 M. It is closed all day on Sundays and holidays.

Members, particularly those from out of town, are cordially invited to use this room on their visits to New York, to have their mail addressed there, and to utilize it as a place for meeting others. There is an ample file of current periodicals, the latest technical books, and the room is well supplied with writing tables.